THE COBWEB THEOREM AND GOVERNMENT POLICY: THE CASE OF KOREAN KIMCHI VEGETABLES

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Since Ezekiel's paper in 1938 [3], many economists have tried to explain price cycles and causal relationships between price and quantity of certain commodities through time. A theoretical tool for explaining them is the cobweb theorem by which the relationship between price and quantity of kimchi vegetables in Korea may also be explained. The problems which existed in the industry of kimchi vegetables in Korea include two aspects: vegetable producers suffer from a very low price from their produce when there is an excess supply, on the other hand, consumers suffer from a very high price when there exists an excess demand for vegetables.

The question is whether there is any way to reduce fluctuations of these vegetable prices. Thus, this paper is to search for an appropriate policy instrument to narrow, to some extent, the magnitude of cobweb cycles of Korean kimchi vegetables.

Traditional Model

Tomek and Robinson in their recent book [8] state that "the cobweb model provides a theoretical explanation of the cyclical component of certain price-quantity paths through time. Prices and quantities are viewed as being linked recursively in a causal chain". According to Ezekiel, Tomek, and Robinson the cobweb model can apply only to commodities which fulfill three conditions: (1) there exists a time lag between the plans and the actual production; (2) producers are price takers and base their production plans on current or recent past prices, hence, the actual production is a function of past prices because of condition (1); (3) current prices are set mainly by current supply available.

The simplest cobweb model developed under the above conditions can be specified by the following three equations:

(1) $q_t^d = \alpha + \beta p_t$ Demand function

(2) $q_t^s = \delta + \gamma p_{t-1}$ Supply function

(3) $q_t^d = q_t^s$ Market equilibrium

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Substituting equations (1) and (2) into (3) and arranging terms:

$$\beta p_t - \gamma p_{t-1} = \delta - \alpha \tag{4}$$

Dividing both sides by β and using polynomials in the lag operator, one has

$$\left(1 - \frac{\gamma}{\beta} L\right) P_t = \frac{\delta - \alpha}{\beta}$$

$$P_t = \frac{\delta - \alpha}{\beta} \cdot \frac{1}{1 - \frac{\gamma}{\beta} L}$$

$$= \beta^{-1}(\delta - \alpha) \sum_{i=0}^{\infty} \left(\frac{\gamma}{\beta}\right)^i L^i$$

$$= \beta^{-1}(\delta - \alpha) \sum_{i=0}^{\infty} \left(\frac{\gamma}{\beta}\right)^i \tag{5}$$

From (5),

if $\left|\frac{\gamma}{\beta}\right| < 1$, P_t will be a convergent cycle;

if $\left|\frac{\gamma}{\beta}\right| > 1$, P_t will be a divergent cycle; and

if $\left|\frac{\gamma}{\beta}\right| = 1$, P_t will be a continuous cycle, as i goes to infinity.

Formulation of Expectations and Random Shocks

Imagine a subsistence farmer who mainly produces vegetables for his own consumption and markets some of them if he has a surplus. This farmer may not be interested in price movement for vegetables because he is not a profit maximizer. Instead he may try to maximize his total production of vegetables. If this is true, then he is a price taker and may consider the previous price at harvest for his plan to produce vegetables. Under the circumstances the classic cobweb model could fit in explaining price cycles for vegetables. However, this is no longer true in many developing countries.

Since the middle of the 1960's the Korean economy has grown rapidly. As a result, many subsistence farmers have been oriented to produce vegetables for commercial use. In the beginning period of this economic growth, the farmers seemed to consider only current prices of vegetables in determining next year's production schedule. However, they soon realized that current prices do not always adequately estimate future prices. Nerlove [5] pointed out the fact that farmers react to expected normal price which is not generally the same as price at harvest. In general, farmers take past prices into account when they plan to produce. Therefore, the current supply of vegetables depends upon past prices, i.e., the effect of a price change is distributed over several periods. Let the supply function of a vegetable be:

$$q_t^s = \delta + \gamma p_t^* \tag{6}$$

Consider the model of expectation formation which was originally developed by Cagan [2] and extensively used in the field of agriculture by Nerlove:

$$p_t^* - p_{t-1}^* = \lambda(p_{t-1} - p_{t-1}^*), \qquad 0 < \lambda \leq 1 \tag{7}$$

where λ is the coefficient of expectations. It says that farmers revise their previous expectations of normal price in each period in proportion to the difference between actual price and what was previously considered to be normal.

Does the realized production of a vegetable always equal the planned production? Obviously, both are not always equal because a variety of random factors influence production. If we carefully observe the price-quantity paths of kimchi vegetables, cycles seemed not to be very smooth with the cobweb model, shown in Figure 1a and 1b. Tomek and Robinson stated that before a cycle can converge or diverge, a random shift in supply starts a new cycle. For instance, unusually favorable weather conditions could result in an unintentionally large supply and a low price, on the other hand, unexpected bad weather would lead to an accidentally small supply and a high price. These low or high prices may have different effects upon production plans for the next period, and consequently, they said, a new cycle would begin. Thus, it is more realistic to have a stochastic supply function:

$$q_t^s = \delta + \gamma p_t^* + u_t \tag{8}$$

where u_t is a random variable and serially uncorrelated.

Lagging equation (8) one period, substituting for p_{t-1}^* in (7), and solving for p_t^*:

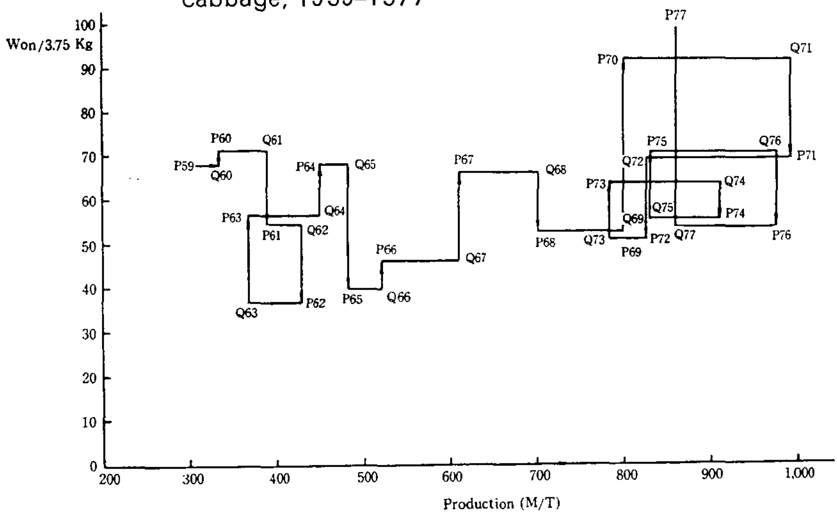
$$p_t^* = (1 - \lambda)\gamma^{-1} (q_{t-1}^s - \delta - u_{t-1}) + \lambda p_{t-1} \tag{9}$$

Substituting (9) into (8):

$$q_t^s = \delta\lambda + \lambda\gamma p_{t-1} + (1 - \gamma)\, q_{t-1}^s + u_t + (\lambda - 1)\, u_{t-1} \tag{10}$$

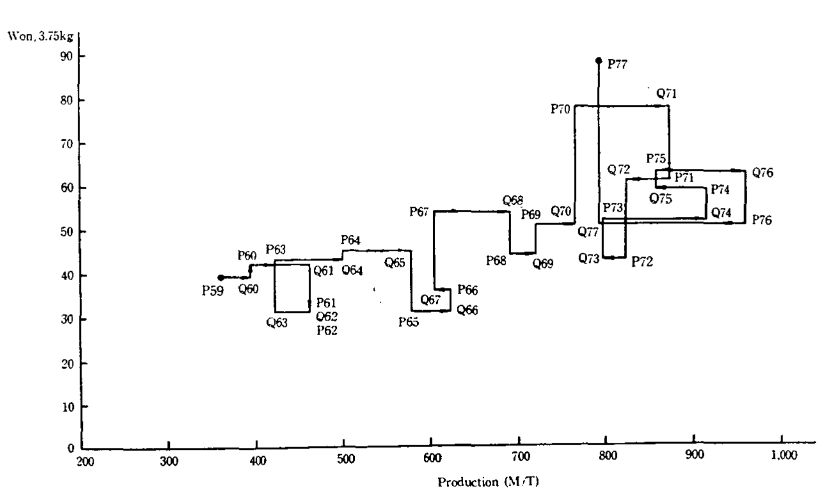
Solving for p_t from the market clearing condition, $q_t^s = q_t^d$ and $q_{t-1}^s = q_{t-1}^d$:

Figure 1a. The relation between price* and production of chinese cabbage, 1959–1977**



* Deflated by the wholesale price indexes, 1970 = 100.
** Based on data reported by both Ministry of Agricculture and Fisheries, and National Agricultural Cooperatives Federation.

Figure 1b. The relation between price* and production of radish, 1959–1977**



* Deflated by the wholesale price indexes, 1970 = 100.
** Based on data reported by both Ministry of Agriculture and Fisheries, and National Agricultural Cooperatives Federation.

$$p_t = \beta^{-1}(\delta - \alpha)\,\lambda + [(\beta^{-1}\gamma - 1)\,\lambda + 1)]p_{t-1} + \beta^{-1}(\lambda - 1)\,u_{t-1} + \beta^{-1}\,u_t$$

or:

$$[1 - ((\beta^{-1}\gamma - 1)\,\lambda + 1)L]\,p_t = \beta^{-1}\,(\delta - \alpha)\lambda + \beta^{-1}(\lambda - 1)\,u_{t-1} + \beta^{-1}\,u_t \tag{11}$$

Dividing both sides by the coefficient of p_t and rearranging it:

$$p_t = \beta^{-1}(\delta - \alpha)\lambda \sum_{i=0}^{\infty} ((\beta^{-1}\gamma - 1)\,\lambda + 1))^i L^i + \beta^{-1}(\lambda - 1) \sum_{i=0}^{\infty} ((\beta^{-1}\gamma - 1)\,\lambda + 1))^i\,u_{t-1-i} + \beta^{-1} \sum_{i=0}^{\infty} ((\beta^{-1}\gamma - 1)\lambda + 1))^i\,u_{t-i} \tag{12}$$

The cobweb cycle would converge or diverge depending upon the following term:

$$|(\beta^{-1}\gamma - 1)\,\lambda + 1| < 1, \qquad \text{(convergence)}$$

$$|(\beta^{-1}\gamma - 1)\,\lambda + 1| > 1, \qquad \text{(divergence).}$$

Note that when the farmer has a shock which is completely different from the previous one, the model would begin a new cycle. From equation (12) one sees that p_t is a function of random shocks.

Counter-Cyclical Policy

So far we have discussed the situation in which there was no government involved in controlling the industry of kimchi vegetables. However, the government might be interested in reducing the amplitude of price cycles under consideration because kimchi is one of the most important foods in Korea.

One may consider a price support program as an alternative for the vegetable industry in Korea. This program might be too expensive if the government wants to purchase and store bulky vegetables because these kimchi vegetables are highly perishable. Then the question is how one can accomplish this. To answer the question assume that the government relies upon the free market system and chooses one of the least expensive tools to make adjustments of the supply of and demand for kimchi vegetables. To make suggestions for a policy tool estimate the expected quantity demanded in time t by plugging the time variable into the demand trend. Denote this estimation with q_t^{d*} which is proportional to the quantity demanded in time $t - 1$, i.e., kq_{t-1}^{d}. It can be the target level of the supply of vegetables.

Suppose the agency desires to minimize the variance of q_t^s around some target level q^*, which is equivalent to q_t^{d*}, by controlling indirectly production areas and/or providing relatively accurate information on expected prices of vegetables. From equation (8) the expected supply is given by

$$Eq_t^s = \delta + \gamma p_t^* = q^* = q_t^{d*} \tag{13}$$

As a feedback rule to inform the farmer of this expected price, the agency has

$$p_t^* = \frac{q_t^{d*} - \delta}{\gamma} \tag{14}$$

Substituting (14) into (8) and deriving the loss function:

$$\text{Var } q = \sigma^2 u. \tag{15}$$

Equations (14) and (15) should be the areas that the government deals with for stabilizing the prices of kimchi vegetables. From equation (14) γ and δ are parameters which can be obtained from the estimated demand and supply functions, we know q_t^d since this is equivalent to the expected quantity of vegetables to be demanded in time t, hence p_t^* can be determined. The government should let the farmers know the expected price p_t^* in advance at least before they make decisions to plant vegetables, so that they can adjust their production such that the sum of individual produce may closely reach to q_t^{d*}. In addition to this effort, the government should try to minimize the variance of vegetable production around the target level q^* by improving the irrigation system, infrastructure, and so forth.

There might be a supplementary policy for balancing both the supply of and demand for kimchi vegetables. The government can organize the farmers, who might be interested in producing vegetables, to register their production plans through cooperatives or some other channels. At the same time, the agency should continue to provide information on an expected price and planned areas so that the farmers are able to adjust their plans for the next production. If the planned area is not desirable with respect to the quantity of vegetables to be demanded, then the agency is required to arrange with some farmers to reduce or expand their production areas as necessary.

Summary and Conclusions

This paper attempts to find an appropriate policy instrument to reduce fluctuations of price cycles for Korean kimchi vegetables, which provide a good example for the cobweb phenomenon.

The classic cobweb model does not fit well in explaining cyclical relationships between price and quantity of certain commodities produced by commercially oriented farmers, simply because they are rational enough to form expectations on future prices. Therefore, it is desirable to take account of expected prices and randon shocks for the model.

With the government involved in controlling the industry of kimchi vegetables, it may be possible to draw a counter-cyclical policy which is actually a feed back rule here in this adjustment mechanism. The government should let the farmers know the expected price in advance so that they can move to adjust their production areas. In addition, the government may be able to minimize the variance of vegetable production around the target level q* through various ways.

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