Journal of Rural Development 2 (November 1979) :113–129

INTEREST RATE EFFECTS ON HOUSEHOLD BEHAVIOR UNDER INFLATIONARY EXPECTATIONS

KIM HAK-UN*

In 1970, Lucas and Rapping, using intertemporal utility and wealth constraint equations, derived a theoretical labor supply function which says that among other things interest rates have an effect on workers' labor supply decisions. Interestingly enough, they also found from their empirical study that the effect is not significant. This difference between theory and fact may exist not only in the labor supply function but also in other household behavioral functions such as the savings or consumption function and the demand for money function when the functions are derived from the intertemporal optimization behavior of the household.

In practice, in his restatement of the quantity theory of money, Friedman (1956) also took pains to point out that the demand for money is influenced by among other things the yield on alternate assets. However, Friedman excluded this yield, the interest rate, from his formulation of the demand for money equation. His exclusion of the interest rate was also based on his empirical results. But his empirical result that the demand for money is independent of the interest rate is largely dependent upon his methodology of analysis. Friedman looked at the correlation of interest rates and residuals derived from a regression of the real quantity of money on permanent income and found none. As David Laidler (1966) has shown using Friedman's data, a multiple regression of the real quantity of money on permanent income and an interest rate has a statistically significant coefficient for the interest rate. Since Laidler's procedure is the correct one, the result is that as far as the demand for money function is concerned, there is no difference between theory and fact. For the savings and consumption functions, the similar problem may arise.

Frequently, this difference between theory and fact in the analysis of the household behavioral function has been reconciled by the simple explanation that the substitution effect is offset by the wealth effect. But, still another difficult question arises: Why does the offset of the two effects take place in one behavioral function such as the labor supply function and not in another function such as the demand for money

* Senior Fellow, Korea Rural Economics Institute, Seoul, Korea.

function, when all behavioral functions are derived from the same wealth constraint and utility functions?

The purpose of this paper is to provide an answer to this question by examining different assumptions regarding the household's portfolio behavior. A main reason for this difference may be found in the treatment of the intertemporal wealth constraint equation, which is closely related to present value analysis. In present value analysis, the interest rate has been the key variable on which the household's optimal allocation of its wealth and income over time is based. It may spend more or less than its current income and borrow or lend the difference in its portfolio. It is therefore expected that its behavior is influenced by the interest rate.

An underlying assumption behind present value analysis which has been totally neglected in the literature is, as will be seen later, that the household does not hold cash balances in its portfolio. The option is either borrowing (supplying bonds) when its receipts are less than its spending, or lending (demanding bonds) in the reverse case. Different assumptions regarding the behavior of the household provide different wealth constraint equations from which different behavioral functions are derived in association with the utility function. Thus, we will examine four cases as follows. In the first case, the household will be assumed to allocate its wealth and income to purchasing bonds, consumption goods, and holding cash balances. In the second case, it will be assumed that the household holds consumption goods and bonds, but not cash balances. In the third case, we will assume that the portfolio of the household consists of consumption goods and cash balances instead of bonds. Finally, we will examine optimal household behavior over time in the process of economic growth. We will see that each case provides a different wealth constraint equation and, hence, different behavioral functions, and conditions the interest rate does not play a key role in determining the behavior of the household will be examined.

1. INTERTEMPORAL ANALYSIS AND THE INTEREST RATE EFFECTS ON THE BEHAVIOR OF HOUSEHOLDS

Let us for simplicity assume that a household's economic horizon spans two periods, period t (the present) and period $t+1$ (the future). We further assume that the household decides all economic decisions, current as well as future, *simultaneously*. This assumption will be used in this section, and will be later replaced by another assumption, the *sequential* decision making assumption. We describe our economy as one where the money supply, M, changes at the rate of u percent per period, so that prices, P, and money wages, W, changes at proper rates of π and ϕ percent, respectively. This actual change in prices and money wages will in turn affect the household's expectations about future prices and money wages with the following definitions:

(1) $$P^*(t+1) = (1+\pi^*(t+1))P(t)$$

(2) $$W^*(t+1) = (1+\phi^*(t+1))W(t)$$

where the starred symbols represent the expected values.

In period t, the household wishes to hold its initial wealth $R(t-1)$, and present income, $Y_N(t)$, in three different forms; consumption goods, $P(t)C(t)$, cash balances, $M(t)$, and bonds (financial claims), $B(t)$. But, when the household holds bonds, it holds them discounted at the going bond rate of interest. Thus the household's wealth constraint in period t is:

(3) $$R(t-1) + Y_N(t) = P(t)C(t) + M(t) + \frac{1}{1+i(t)} B(t)$$

where $P(t)$ is the price level of consumption goods in period t and $i(t)$ is the bond rate of interest in period t.

In period $t+1$, the household expects to receive back from holdings in discounted bonds a gross return which is the sum of the original discounted amount and its interest income:

(4) $$B(t) = \frac{1}{1+i(t)} B(t) + \frac{i(t)}{1+i(t)} B(t).$$

And also, the household will transfer the money stock held in period t to this period. Therefore, the household will spend its entire portfolio, $M(t) + B(t)$, and its expected income in period $t+1$, $Y_N^*(t+1)$, for the purpose of expenditures in period $t+1$, $E^*(t+1)$, which is further divided into the three different forms; consumption goods, cash balances, and bonds discounted at the expected bond rate of interest:

(5) $$\begin{aligned} M(t) + B(t) + Y_N^*(t+1) &= E^*(t+1) \\ &= P^*(t+1)C^*(t+1) + M^*(t+1) \\ &\quad + \frac{1}{1+i^*(t+1)} B^*(t+1) \end{aligned}$$

where the starred symbols stand for the expected values. (5) is the wealth constraint in period $t+1$. From this wealth constraint equation it is apparent that the initial wealth $R(t-1)$ in (3) is the sum of the initial bonds held by the household and initial money stock in the past such that

(6) $$R(t-1) = M(t-1) + B(t-1).$$

Substituting (5) into (3) with respect to B(t) will yield an intertemporal wealth constraint equation:

(7) $$\begin{aligned} R(t-1) &+ Y_N(t) + \frac{1}{1+i(t)} Y_N^*(t+1) \\ &= P(t)C(t) + \frac{1}{1+i(t)} E^*(t+1) + \frac{i(t)}{1+i(t)} M(t). \end{aligned}$$

The terms in the left-hand side of (7) represent the present value of sources of wealth, while the terms in the right-hand side the present value of uses of wealth. In addition, we have the Fisherine relationship:

$$(8) \qquad (1 + i(t)) = (1 + \rho(t))\,(1 + \pi^*(t+1)),$$

where $\rho\,(t)$ is the real bond rate of interest in period t.

In equation (7), all the terms are expressed in nominal terms. Let us now change them into real terms by dividing through by $P(t)$, and using (1), (2), and (8) as follows:

$$(9) \qquad \frac{R(t-1)}{P(t)} + y_N(t) + \frac{1}{1+\rho(t)} y_N^*(t+1)$$
$$= C(t) + \frac{1}{1+\rho(t)} e^*(t+1) + \frac{i(t)}{1+i(t)} \frac{M(t)}{P(t)}$$

where $y_N(t) = Y_N(t)/P(t)$, current real income, and $y_N^*(t+1) = Y_N^*(t+1)/P^*(t+1)$, expected real income, and $e^*\,(t+1) = E^*\,(t+1)/P^*(t+1)$, the expected real expenditures.

Since interest income from holdings of bonds by the household is separately expressed as $iB/(1+i)$ in (5) via the relationship in (4), the income, Y_N, can be considered only as its money wage income, the money wage rate, W, multiplied by hours worked, N, so that

$$(10) \quad Y_N(t) = W(t)N(t) \text{ and } Y_N^*(t+1) = W^*(t+1)\,N^*(t+1)$$

or in real terms

$$(11) \qquad y_N(t) = W(t)N(t)/P(t) \text{ and}$$
$$y_N^*(t+1) = W^*(t+1)N^*(t+1)/P^*(t+1).$$

Then the optimization problem applies to the household which is assumed to maximize a utility index

$$(12) \quad U = U(\,N(t), C(t), M(t)/P(t), B(t)/P(t), N^*(t+1), E^*(t+1))$$

subject to wealth constraint equations (9) and (11). We have therefore the household's behavioral functions:

$$(13) \quad M(t)/P(t) = L(y_N(t), y_N^*(t+1), \rho(t), \pi^*(t+1), R(t-1)/P(t))$$

$$(14) \qquad C(t) = C(y_N(t), y_N^*(t+1), \rho(t), \pi^*(t+1), R(t-1)/P(t))$$

$$(15) \qquad N(t) = H(w(t), w^*(t+1), \rho(t), \pi^*(t+1), R(t-1)/P(t))$$

where $w(t) = W(t)/P(t)$, the real wage rate in period t, and $w^*(t+1) = W^*(t+1)/P^*(t+1)$, the expected real wage rate in period $t+1$. Two features are worthy of note. First, notice that instead of division of $E^*(t+1)$ between $C^*(t+1)$ and other portfolio assets, $E^*(t+1)$ itself enters into the utility function and the wealth constraint function. That is, unlike current expenditures, future expected expenditures are not broken down into the three different forms of wealth. The reason is that it would be foolish, in forming expectations, to specify to a greater extent matters which are very uncertain. It is reasonable, therefore, to be guided to a considerable degree by the facts about which we feel somewhat confident, even though they may be less relevant to the issue than other facts about which our knowledge is vague.

Second, real balances are introduced into the utility function. An issue raised here is to what extent should they be introduced? This issue was discussed excellently by Patinkin (1966, Chapter V:1). He says that the introduction of uncertainties via the stochastic payment process readily enables us to analyze the problem of holding money by means of the traditional calculus (p.81). Samuelson (1968) also introduces money into the utility function because, as he writes, "... one can put M into the utility function, along with other things, as a real convenience in a world of stochastic uncertainty and invisible transaction charges." (p.8). It has been generally argued that real balances give utility because money in real terms facilitates the process of transactions, bridges gaps between payments and receipts, makes unexpected purchase possible, and so on. These kinds of usefulness justify the incorporation of real balances into the utility function.

1–1. Experiment 1

The behavioral functions given above are so general that nearly everyone may accept them on a purely formal and abstract level, although each would choose to express them differently in detail. Thus assume first that alternative forms of holding wealth are consumption goods and bonds. That is, it is assumed that the household does not hold cash balances but holds bonds and consumption goods in its portfolio. Then the optimization problem is to maximize utility so that

$$U = U(N(t), N^*(t+1), C(t), E^*(t+1)) \tag{16}$$

subject to

$$\frac{R(t-1)}{P(t)} + y_N(t) + \frac{1}{1+\rho(t)} y_N^*(t+1) = C(t) + \frac{1}{1+\rho(t)} e^*(t+1) \tag{17}$$

and equation (11). Equations (16) and (17) are different from equations (12) and (9), respectively, in that real balances are zero. Nevertheless, equations (16) and (17) have been the standard forms of intertemporal optimization of a household in the literature. In fact, equation (17) is the same wealth constraint equation which many works provide for present value analysis. It becomes apparent that they have been mistakenly based on the implicit assumption that the household does not hold cash balances but holds bonds and consumption goods in its portfolio.

The conditions for utility maximization will yield

(18) $$M(t)/P(t) = 0$$

(19) $$C(t) = C(y_N(t), y_N^*(t+1), \rho(t), R(t-1)/P(t))$$

(20) $$N(t) = N(w(t), w^*(t+1), \rho(t), R(t-1)/P(t)).$$

It is important to notice that the real bond rate of interest remains as an argument in the functions, while the expected rate of price change is dropped, even though prices are from the beginning expected to rise in the next time period, as shown in (5). This is largely due to the absence of the demand for cash balances. Since the household does not demand cash balances in this case, there is no need for the household to signal inflationary expectations by drawing on cash balances and savings account and rearranging its portfolio. When we think that the assumption has been totally ignored and is far from realistic and moreover is not compatible with the existence of the demand for money function, it is surprising that equation (17) has thus far been the most popular form of the wealth constraint equation and that the resultant behavioral functions have been used along with the demand for money function within the general equilibrium model without any consideration of the assumption.

1–2. Experiment 2

Alternatively, let us assume that holdings of bonds by the household are zero, but holdings of cash balances are not zero. This implies that the only alternative to holding cash balances is to hold consumption goods. This assumption was made by Cagan (1956). Observing seven hyperinflations in six countries, Cagan noticed that the interest rate effect on the demand for money was minor. Cagan therefore suggested a hypothesis that the change in demand for money must be due to a change in some other variable—— namely the expected rate of price change. He reasoned that

> The money return on bonds includes interest and on equities includes dividends, as well as any gains or losses due to a change in the money values of the assets. Variations in the cost of holding cash balances when the alternative is to hold consumers' goods can be determined solely by the change in the real value of a given nominal cash balance. [Cagan 1956, p. 31]

Deaver (1961) also explains this by referring to the poorly developed markets for investment, which limit the ability of people to hold their money in alternative froms. Such a state of poorly developed markets may be prevalent in many less-developed countries.

Under this assumption, let $B(t)$ be zero and substitute the expression for $M(t)$ implicit in (5) into (3). Then we obtain another wealth constraint equation:

$$\text{(21)} \qquad \frac{R(t-1)}{P(t)} + y_N(t) + (1 + \pi^*(t+1))y_N^*(t+1) = C(t) + (1 + \pi^*(t+1))e^*(t+1).$$

Here the striking result is that the bond rate of interest does not appear as an argument even in the intertemporal wealth constraint equation. Associated with this wealth constraint and (10), the household's behavioral functions are given by

$$\text{(22)} \qquad M(t)/P(t) = L(y_N(t), y_N^*(t+1), \pi^*(t+1), R(t-1)/P(t))$$

$$\text{(23)} \qquad C(t) = C(y_N(t), y_N^*(t+1), \pi^*(t+1), R(t-1)/P(t))$$

$$\text{(24)} \qquad N(t) = H(w(t), w^*(t+1), \pi^*(t+1), R(t-1)/P(t)).$$

These functions are sharply different from the previous results in that the real bond rate of interest which appears before is now replaced by the expected rate of price change.

An economic interpretation of two different wealth constraints (17) and (21), and associated behavioral functions is the following. In so far as the household spends some of its wealth and income on purchasing bonds for the interest receipts, the rate of return of holding bonds is of course the bond rate of interest it expects to receive. As a holder of bonds, the household is now sensitive to the movement of the bond rate of interest.

If, on the other hand, it does not spend any of its wealth and income to purchase bonds but spends them for the purchase of consumption goods and keeps the remainder in the form of cash balances for precautionary motives, it would not be interested in fluctuations of the bond rate of interest. In this case, the expected rate of price change alone is its concern because it is the opportunity cost of holding money. Furthermore, the household would not know the real bond rate of interest and it may cost it something to collect that information. Hence its current economic behavior can be described by functions (22) through (24) which state that the behavior of the household is determined by current real income, future expected income, the expected rate of price change, and initial real wealth.

Such a description of behavior of the household as determined by the expected rate of price change, instead of the real bond rate of interest, would disturb readers who regard it as a special case under the assumption that the household does not hold wealth in the form of bonds and ask what good is the argument based on such a special case. It would be unfair to interpret it in such a way because, first, it would be more difficult to imagine a situation where the household does not hold cash balances so that the behavioral functions amount to (18) through (20) than a situation where it does not hold bonds so that the functions become (22) through (24). Second, there have been empirical findings that the household's optimum behavior would not significantly be affected by the real rate of interest, but by the expected rate of price change [see for example Lucas and Rapping (1970)].

This investigation provides at least a theoretical explanation for this empirical finding. Finally, it can be extended without substantial changes in the result to the case where the household holds not only cash balances but also some assets (bonds) whose yields are anticipated with almost perfect certainty. These assets are characterized by high liquidity and close substitution for money. There is no capital losses attached to these assets, the behavioral functions amount to ones including the interest rate as an argument. However, since their yield rates are considered to be constant, they do not affect economic decisions of the household. It should be emphasized, therefore, that the demand for money is almost limited to transaction and precautionary demands because the household prefers to hold assets free from capital losses. Who demands money (or supplies bonds) for speculative motives in this circumstance? Only relatively small group of speculators whom we do not think of as households.

2. INTERTEMPORAL ANALYSIS UNDER SEQUENTIAL DECISION MAKING ASSUMPTION

In the above, the desirable result came out at the cost of the special assumption regarding the portfolio behavior of the household, though the assumption was justified in many ways. However, the result is still very dependent upon those strong assumptions regarding portfolio behavior. A more ambitious attempt must therefore be made with a less strict and more reasonable assumption.

An implicit assumption in the previous section was that all economic decisions are simultaneously made by the household. Instead of this *simultaneous* decision making, let us use a *sequential* decision making process, following the neo-Keynesian position. The neo-Keynesian position may be summarized as follows, although we do not find any clear summary of it in any single work.

It is assumed that the household in initial equilibrium decides first the optimum allocation of its time between labor supply to earn income and leisure for pleasure. Then it follows a sequential process whereby it divides its income between saving and consumption. Saving is later divided into transaction balances and other portfolio assets which themselves are further divided into speculative cash balances and non-cash investment and the final step of this sequence is the allocation of the remaining saving among alternative non-money assets. The implicit assumption here is that this process is recursive and that each decision is independent of the subsequent decisions.

2–1. The Labor Supply Function

With this neo-Keynesian assumption of the sequential process, we should consider the wealth constraint and utility functions again. With reference to the definition of $R(t-1)$ in (6), we can rewrite (3) and (5) as:

$$(25) \qquad M(t-1) + B(t-1) + Y_N(t) = P(t)C(t) + M(t) + B(t)/(1 + \mathbf{i}(t))$$

$$(26) \qquad M(t) + B(t) + Y_N^*(t+1) = P^*(t+1)C^*(t+1) + M^*(t+1) + B^*(t+1)/(1 + \mathbf{i}(t+1)).$$

According to the sequential decision making, the household decides first the total amount of portfolio before it breaks it down into money and bonds. Thus let us define the value of the household's portfolio as the value of the composite good A (See Patinkin 1966, p.105):

$$(27) \qquad A = M + B/(1 + \mathbf{i})$$

and define

$$(28) \qquad (1+i)A = M + B$$

so that

$$(29) \qquad i = (M + B)/A - 1 = \mathbf{i}((1 - M/A) + O(M/A).$$

That is, i is the weighted average of the rates of return on bonds ($\mathbf{i}$) and money (0), and must therefore lie between these two values. The wealth constraint equations can then be given by

$$(30) \qquad (1 + i(t-1))A(t-1) + Y_N(t) = P(t)C(t) + A(t)$$

$$(31) \qquad (1 + i(t))A(t) + Y_N^*(t+1) = P^*(t+1)C^*(t+1) + A^*(t+1) = E^*(t+1).$$

Substituting (31) into (30) with respect to $A(t)$ will produce

$$(32) \qquad (1 + i(t-1))A(t-1) + Y_N(t) + \frac{1}{1 + i(t)} Y_N^*(t+1) = P(t)C(t) + \frac{1}{1 + i(t)} E^*(t+1)$$

which is equivalent to

$$(33) \quad (M(t-1) + B(t-1))/P(t) + W(t)N(t) + w^*(t+1)N^*(t+1)/(1+r(t)) = C(t) + e^*(t+1)/(1 + r(t))$$

since

$$(1 + i(t)) = (1 + r(t))(1 + \pi^*(t+1)) \tag{34}$$

where r is the real rate of interest which is the weighted average of the real bond rate of return on bonds (ρ) and money (0), such that

$$r = \rho(1-M/A) + 0(M/A).$$

Definition (34) represents the Fisherine effect expressed in terms of the weighted average rate.

From the beginning we have dealt with the economy where the money supply changes at the rate of u percent per period so that prices and money wages change at the rate of π percent and ϕ percent, respectively. The effect of a rise in the rate of money supply is absorbed partly by a rise in the rate of price change and partly by a rise in the rate of real income, g. Therefore, as we assumed previously that prices and money wages are expected to rise by π^* percent and ϕ^* percent, respectively, so we now assume that the household expects that the level of expenditures will rise by g^* percent, along with the expected rise in labor supply by g^* percent, such that

$$e^*(t+1) = (1 + g^*(t+1))e(t) \tag{35}$$

$$N^*(t+1) = (1 + g^*(t+1))N(t). \tag{36}$$

It may be worth stressing the importance of quantity expectations as well as price expectations in the study of inflation and growth. Following this spirit, we introduce explicitly quantity expectations.

Substituting (34) through (26) into (33) will give us

$$\begin{aligned} \frac{R(t-1)}{P(t)} + w(t)N(t) + \frac{1 + g^*(t+1)}{1 + r(t)} w^*(t+1)N(t) \\ = C(t) + \frac{1 + g^*(t+1)}{1 + r(t)} e(t) \\ = D(t). \end{aligned} \tag{37}$$

In association with the household's utility function:

$$U = U(N(t), \quad D(t)), \tag{38}$$

one may end up with the following labor supply function:

$$N(t) = H(w(t), w^*(t+1), R(t-1), (1 + g^*(t+1))/(1 + r(t))) \tag{39}$$

where $R(t-1) = (M(t-1) + B(t-1))/P(t)$,
represents the initial real wealth.

On a purely theoretical level, von Neumann's (1945) general equilibrium growth model shows that in equilibrium growth the real rate of interest is equal to the rate of growth. It follows that $(1 + g^*(t + 1))/(1 + r(t)) = 1$ so that the labor supply function now is

$$N(t) = H(w(t), w^*(t+1), R(t-1)). \tag{40}$$

In practice, Friedman assumed that the difference between the expected rate of growth and the real rate of interest, $r - g^*$, is regarded as constant, determined outside the system. He reasoned that

> (1) that over a time interval relevant for the analysis of short-period fluctuations, r and g^* can be separately regarded as constant; (2) that the two can be regarded as moving together, so that difference will vary less than either. Of course, in both cases, what is relevant is not absolute constancy, but changes in r-g* that are small compared to changes in π^*. [Friedman 1971, p. 36. My notation is substituted for his.]

On this basis, the labor supply function is given by (40). Notice that the rate of interest does not enter the function as an argument. This implies that the household's labor supply decisions will not be determined by the interest rate.

In order to see the sign of each variable, first of all, one may construct the Slutsky equations

$$\left.\frac{dN(t)}{dw(t)}\right|_{dw^*(t+1)=0} = \left.\frac{dN(t)}{dw(t)}\right|_{dU=0} - N(t)\left.\frac{dN(t)}{dR(t-1)}\right|_{dw(t)=0} \tag{41}$$

$$\left.\frac{dN(t)}{dw^*(t+1)}\right|_{dw(t)=0} = \left.\frac{dN(t)}{dw^*(t+1)}\right|_{dU=0} - N(t)\left.\frac{dN(t)}{dR(t-1)}\right|_{dw^*(t+1)=0}. \tag{42}$$

For convenience sake, let us simplify the above by using equivalent notations:

$$H_1 = s_1 - N(t)H_3 \tag{43}$$

$$H_2 = s_2 - N(t)H_3. \tag{44}$$

H_1 and H_2 are the direct total effect and the cross total effect, respectively. s_1 and s_2 indicate the direct substitution effect and the cross substitution effect, respectively. H_3 is the wealth effect. The signs of H_1 and H_2 are in general uncertain as one would expect. To know them let us consider the following.

First, there is a famous theorem that all goods cannot be complements to each other. Mathematically, in the case of two goods, it is expressed by $p_1s_1 + p_2s_2 = 0$ where p_1 and p_2 are prices of good 1 and good 2, respectively. When we apply this theorem to our analysis, it implies that future leisure is a substitute for current leisure and that the direct substitution effect s_1 in (43) is always positive, the cross substitution effect s_2 in (44) must be negative.

Second, the wealth effect H_3 may be of either sign. The total effect of a wage change on the supply of labor is thus uncertain. However, if we assume that leisure is not inferior, then $H_3 < 0$. From this it follows that in (43)

$$H_1 > 0. \tag{45}$$

Third, since leisure is not a Giffen good, the negative wealth effect is not large enough to offset the negative cross substitution effect. Thus, we have in (44)

$$H_2 < 0. \tag{46}$$

The Slutsky equations may be expressed in another equivalent form. Multiplying (43) and (44) through by $w(t)$ and $w^*(t+1)$, respectively, and summing the results, we have

$$\begin{aligned}(47)\quad & w(t)H_1 + w^*(t+1)H_2 \\ & = w(t)s_1 + w^*(t+1)s_2 - w(t)N(t)H_3\, w^*(t+1)N(t)H_3.\end{aligned}$$

Since $w(t)s_1 + w^*(t+1)s_2 = 0$, this can be reduced to

$$\begin{aligned}(47)\qquad & w(t)H_1 + w^*(t+1)H_2 \\ & = -(w(t)N(t) + w^*(t+1)N^*(t+1))H_3 > 0.\end{aligned}$$

The result in (47) may also be supported by the theorem that the own price elasticity is bigger than the cross price elasticity.

We can modify (40) in such a way that it gives the current labor supply as a function of independent variables such as $w(t)$, $\phi^*(t+1)$, $\pi^*(t+1)$, and $R(t-1)$ since $w^*(t+1) = w(t)\exp(\phi^*(t+1) - \pi^*(t+1))$. The total differential of (40) is

$$dN(t) = H_1 dw(t) + H_2 dw^*(t+1) + H_3 dR(t-1).$$

Since

$$\begin{aligned} dw^*(t+1) &= \exp(\phi^*(t+1) - \pi^*(t+1))dw(t) + w(t) \\ & \exp(\phi^*(t+1) - \pi^*(t+1))(d\phi^*(t+1) - d\pi^*(t+1)),\end{aligned}$$

it is equivalent to

$$\begin{aligned} dN(t) &= (H_1 + H_2 Z)dw(t) + H_2 w(t) Z\, d\phi^*(t+1) \\ & - H_2 w(t) Z\, d\pi^*(t+1) + H_3 dR(t-1),\end{aligned}$$

where $Z = \exp(\phi^*(t+1) - \pi^*(t+1))$. Therefore, we have an equivalent for (40):

$$N(t) = h(w(t), \phi^*(t+1), \pi^*(t+1), R(t-1)) \tag{48}$$

where

$$(49)\qquad h_1 = H_1 + H_2 Z = (w(t)H_1 + w^*(t+1)H_2)/w(t) > 0 \quad \text{from} \quad (47)$$

$$(50)\qquad h_2 = H_2 w(t) Z < 0$$

$$(51)\qquad h_3 = -H_2 w(t) Z > 0$$

$$(52)\qquad h_4 = H_3 < 0.$$

From (50) and (51) it follows that

$$(53)\qquad h_2 + h_3 = 0.$$

In an alternative way, equation (48) can be obtained directly from (37). Since $w^*(t+1) = w(t)Z$ or $w^*(t+1) = w(t)(1 + \phi^*(t+1))/(1 + \pi^*(t+1))$, the wealth constraint equation (37) can be written as:

$$R(t-1) + w(t)N(t) + \frac{(1 + g^*(t+1))}{(1 + r(t))} \frac{(1 + \phi^*(t+1))}{(1 + \pi^*(t+1))} w(t)N(t) = C(t) + \frac{(1 + g^*(t+1))}{(1 + r(t))} e(t).$$

The labor supply function derived in association with this wealth constraint will be the same as (48).

2–2 The Consumption and Saving Functions

After the household earns its income, it will at the second stage divide the income between saving and consumption. To see this second process, let us reproduce equations (30) and (31):

$$(54)\qquad (1 + i(t-1))A(t-1) + W(t)N(t) = P(t)C(t) + A(t)$$

$$(55)\qquad (1 + i(t))A(t) + W^*(t+1)N^*(t+1) = P^*(t+1)C^*(t+1) + A^*(t+1).$$

Dividing (54) by $P(t)$ and (55) by $P^*(t+1)$ will yield

$$(56)\qquad \frac{A(t-1)}{P(t-1)} + r(t-1)\frac{A(t-1)}{P(t-1)} + w(t)N(t) = C(t) + \frac{A(t)}{P(t)}$$

$$(57)\qquad \frac{A(t)}{P(t)} + r(t)\frac{A(t)}{P(t)} + w^*(t+1)N^*(t+1) = C^*(t+1) + \frac{A^*(t+1)}{P^*(t+1)}$$

with the aid of (34). Let us define total real income, $y(t)$, as the sum of wage income and interest income:

$$(58)\qquad y(t) = w(t)N(t) + r(t-1)\frac{A(t-1)}{P(t-1)} = y_N(t) + r(t-1)\frac{A(t-1)}{P(t-1)}.$$

Thus, (56) and (57) become

(59) $$y(t) = C(t) + \left(\frac{\dot{A}}{P}(t)\right)$$

(60) $$y^*(t+1) = C^*(t+1) + \left(\frac{\dot{A}^*}{P^*}(t+1)\right)$$

where the dot on (A/P) indicates the change in portfolio, which is by definition saving, such that

(61) $$S(t) = \left(\frac{\dot{A}}{P}(t)\right) = \frac{A(t)}{P(t)} - \frac{A(t-1)}{P(t-1)}$$

(62) $$S^*(t+1) = \left(\frac{\dot{A}^*}{P^*}(t+1)\right) = \frac{A^*(t+1)}{P^*(t+1)} - \frac{A(t)}{P(t)}.$$

It is important to notice in (59) and (60) that we choose to measure total real income, y, in consumption units so that the same price unit can be applicable to C and y.

Divide equation (60) by $1/(1+r(t))$ and add the result to equation (59). Then we have

(63) $$y(t) + \frac{1}{1+r(t)}y^*(t+1) = C(t) + \frac{1}{1+r(t)}C^*(t+1) + S(t) + \frac{1}{1+r(t)}S^*(t+1).$$

Again, recalling the importance of quantity expectations in our analysis, let us adopt the following definitions:

$$y^*(t+1) = (1+g^*(t+1))y(t)$$
$$C^*(t+1) = (1+g^*(t+1))C(t)$$
$$S^*(t+1) = (1+g^*(t+1))S(t).$$

We then have

(64) $$y(t) + \frac{(1+g^*(t+1))}{(1+r(t))}y(t) = C(t) + \frac{(1+g^*(t+1))}{(1+r(t))}C(t) + S(t) + \frac{(1+g^*(t+1))}{(1+r(t))}S(t)$$

which is now reduced to a simple budget constraint equation:

(65) $$y(t) = C(t) + S(t).$$

Together with the utility function in this second stage,

$$U = U(C(t), S(t)),$$

equation (2–65) will generate the consumption and saving functions:

(66) $$C(t) = C(y(t))$$

(67) $$S(t) = S(y(t)).$$

2–3 The Assets Demand Functions

At the third stage, the household divides saving into money and bonds. Its utility function at this stage is

$$U = U\left(\left(\frac{\dot{M}}{P}(t)\right), \left(\frac{\dot{B}}{P}(t)\right)\right)$$

and its budget constraint is

$$\begin{aligned} S(t) &= \frac{A(t)}{P(t)} - \frac{A(t-1)}{P(t-1)} \\ &= \left(\frac{\dot{M}}{P}(t)\right) + \frac{1}{1+i(t)}\left(\frac{\dot{B}}{P}(t)\right). \end{aligned}$$

The optimization problem will therefore end up with

(68) $$\begin{aligned} \left(\frac{\dot{M}}{P}(t)\right) &= L(S(y(t)), i(t)) \\ &= L(y(t), \rho(t), \pi^*(t+1)) \end{aligned}$$

(69) $$\begin{aligned} \left(\frac{\dot{B}}{P}(t)\right) &= B(S(y(t)), i(t)) \\ &= B(y(t), \rho(t), \pi^*(t+1)) \end{aligned}$$

with the following restriction on partial derivatives:

$$\frac{\partial S}{\partial \rho} = \frac{\partial L}{\partial \rho} + \frac{\partial B}{\partial \rho} = 0$$

and

$$\frac{\partial S}{\partial \pi^*} = \frac{\partial L}{\partial \pi^*} + \frac{\partial B}{\partial \pi^*} = 0$$

which are reduced to

$$L_2 = -B_2 < 0$$

and

$$L_3 = -B_3 < 0.$$

This relationship is supported by the gross substitution assumption between money and bonds that an increase in the rate of return on bonds raises the quantity demanded of bonds and lowers the quantity of money demanded of money.

Since $M^d(t-1) = M(t-1)$ and $B^d(t-1) = B(t-1)$ in the past period, equations (68) and (69) can be written in terms of stock instead of flow:

(2–70) $$m^d(t) = 1(y(t), \rho(t), \pi^*(t+1), m(t-1))$$

(2–71) $$b^d(t) = b(y(t), \rho(t), \pi^*(t+1), b(t-1))$$

where $m = M/P$ and $b = B/P$, with the following restrictions on partial derivatives:

$$l_1 > 0, \quad l_2 < 0, \quad l_3 < 0$$
$$b_1 > 0, \quad b_2 > 0, \quad b_3 > 0$$

and

$$l_2 + b_2 = 0$$
$$l_3 + b_3 = 0.$$

In the earlier discussion of this paper, we were aware of the difference between theory and fact regarding the interest rate effects on the household behavior. We discucssed that this difference between theory and fact may exist in the household behavioral functions when the functions are derived from the intertemporal optimization behavior of the household behavior. We continued to discuss that this difference may be explained by the offset phenomenon of the substitution and wealth effects. But we noticed that this explanation has one defect; it provides a justification for the interest rate effect on the household's labor supply decisions but does not for the interest rate effects on other behavioral functions.

We can see that the entire problem is related to the assumption regarding the household's portfolio behavior. As we saw, difference between theory and fact may arise because of the simultaneous decision making assumption. On the other hand, the sequential decision making assumption makes theory agree with fact through the entire household behavioral functions.

SUMMARY

The purpose of this paper has been to show the theoretical derivation of all the household's behavioral functions under inflationary expectations. To do this successfully, we discussed the general problems of intertemporal analysis, exploiting the implications of assumptions hidden behind the wealth constraint equations and the utility equations, which are supposed to reflect the behavior of the household. It was found that whether the real rate of interest plays a key role in determining economic decisions of the household depends upon behavioral assumptions regarding the portfolio.

REFERENCES

Cagan, P., 1956, "The Monetary Dynamics of Hyperinflation," in *Studies in the Quantity Theory of Money*, edited by M. Friedman, Chicago: University of Chicago Press.

Deaver, J.V., 1961, *The Chilean Inflation and the Demand for Money* (unpublished Ph. D. dissertation), Chicago: University of Chicago.

Friedman, M., 1956, "The Quantity Theory of Money-Restatement," in *Studies in the Quantity Theory of Money*, edited by M. Friedman, Chicago: University of Chicago Press.

———, 1971, "A Monetary Theory of Nominal Income," *Journal of Political Economy* 79 (March/April): 323–337.

Laidler, D., 1966, "The Rate of Interest and the Demand for Money—Some Empirical Evidence," *Journal of Political Economy* 74 (December): 545–555.

Lucas, R. E., and L. A. Rapping, 1970, "Real Wages, Employment, and Inflation," in *Microeconomic Foundations of Employment and Inflation Theory*, edited by E.S. Phelps, New York: Norton.

von Neumann, J., 1945, "A Model of General Economic Equilibrium", *Review of Economic Studies* 13 (1945–46): 1–9.

Patinkin, D., 1965, *Money, Interest, and Prices*, New York: Harper & Row.

Samuelson, P., 1968, "What Classical and Neoclassical Monetary Theory Really Was," *Canadian Journal of Economics* 1 (February): 1–15.

빈 면