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# TOWARDS DERIVING DEMAND FUNCTIONS FOR PUBLIC GOODS

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## I. Introduction

One may observe that many forms of agricultural outputs and aspects of the rural environment are public goods. The Saemaul movement, for example, may be best understood as an economic activity using inputs to produce public goods as well as private goods in rural communities. Despite this fact, there have been no attempts to study agricultural economics from this approach, nor is there a tendency to catch the spirit of the problem in such a way. This may be either because agricultural economists have not been aware of this point or because there are some fundamental difficulties in the study of public goods. Whatever the reason, the importance of public goods in agricultural economics cannot be overlooked.

One of the fundamental difficulties in the study of public goods in general is that because there are no markets for public goods, consumer perferences are not revealed and, hence, the demand for them cannot be measured in the ordinary price-output space as for private goods. Therefore, other ways of measuring consumer preferences and defining the demand for public goods must be investigated.

One of them, probably the most natural approach, is to ask each individual how much he is willing to pay for public goods provided by communities, expecting him to reveal his true preference and, hence, his demand for public goods. Since Wicksell, it has long been recognized that each individual may have strong incentives to misrepresent or not disclose his true preferences for public goods, to his advantage and at the expense of others in the community. This raises the question as to whether it is possible to find the magnitude of this bias.

The purpose of this paper is to search for an answer to this question. The primary concern is to derive an individual's true demand function for public goods provided by the community. For this, the paper surveys the me- thods presented in the literature particularly, Maler's analysis

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and Bowen's voting procedure. A new method is then discussed.

## II. Definitions

D1. *Individual Demand Price of Public Goods:* Let us refer to an economy with a private good X and a public good Y. The price of the private good is P which is uniform to all consumers in the economy. On the other hand, the *true* price of the public good is by definition different from one individual to another insofar as each individual's preference for the public good is different, and is unknown because each individual's true prerence for a public good is not truly revealed. Instead, we observe only the *stated* demand price of the public good of each individual. Let individual *i*'s *stated* demand price of the public good Y be $Q_i$.

D2. *Total Demand Price of Public Goods:* Should there be *m* individuals in this economy, the total *stated* demand price of the public good Y is given by

$$Q = \sum_{i=1}^{m} Q_i.$$

D3. *Compensated Demand Function:* The Compensated demand function for the private good X is defined as:

$$X = X(P, Y, \bar{U})$$

where $\bar{U}$ is a constant utility level.

D4. *Expenditure Function:* The expenditure function for the private good $X$ is defined as:

$$\begin{aligned} E &= PX \\ &= E(P, Y, \bar{U}). \end{aligned}$$

D5. *Consumer Surplus:* Suppose that we were at an initial situation with the price of the private good P′ and the supply of the public good Y′. Suppose further that we are interested in measuring the welfare effects of a change in the economy from this initial situation to a new situation where the price of the private good is P″ and the supply of the public good is Y″. Before the change, the initial demand for the private good was given by $X' = X(P', Y,' \bar{U})$ and the initial expenditure function for the private good was $E'(P', Y', \bar{U})$. Let us define this initial expenditure $E'(P', Y', \bar{U}) = I$. After the change, for consumer to be on the same utility level $\bar{U}$, the expenditure of the consumer must be $E''(P'', Y'', \bar{U})$. Then the consumer surplus measure of welfare change in monetary terms can be defined as:

$$\begin{aligned} S &= E'(P', Y', \bar{U}) - E''(P'', Y'', \bar{U}) \\ &= I - E''(P'', Y'', \bar{U}). \end{aligned}$$

## III. Demand Prices for Public Goods

Consider individual i facing a set A in the $(X, Y)$ space. The set consists of all of pairs $(X, Y)$ which yields utility equal to $\bar{U}$. Take a point $(X', Y')$ in this set where $X' = X(P, Y', \bar{U})$ is not an interior point to A because otherwise $X'$ would not be maximized on $A(Y')$. Then, there exists a supporting hyperplane to A at $(X', Y')$ with its normal $(P, Q_i)$ such that

(1) $P(X - X') + Q_i(Y - Y') = 0.$

In the same manner, we can have at another point $(X'', Y'')$ on A:

(2) $P(X - X'') + Q_i(Y - Y'') = 0.$

Subtracting (2) from (1) will yield

(3) $P(X' - X'') + Q_i(Y' - Y'') = 0.$

Using the definition of the expenditure funcgion for individual $i$ shown in the above, (3) is equivalent to

$$E_i'(P, Y', \bar{U}) - E_i''(P, Y'', \bar{U}) + Q_i(Y' - Y'') = 0$$

which becomes

$$-Q_i(Y' - Y'') = E_i'(P, Y', \bar{U}) - E_i''(P, Y'', \bar{U}).$$

This will reduce to

$$-Q_i \Delta Y = \Delta E_i.$$

Therefore,

$$\lim_{\Delta \to 0} - \frac{\Delta E_i}{\Delta Y} = - \frac{\partial E_i}{\partial Y} = Q_i.$$

This implies that individual i's "stated" demand price of the public good $Y$ can be interpreted as the partial derivative of his expenditure function with respect to the public good $Y$ with opposite sign.

## IV. Demand for Public Goods

Now it is time to discuss the bias of the revealed preference for public goods, using definitions and concepts developed in the above. Our point of departure is Bowen's voting procedure.

1. Bowen (1943) showed that an individual's preference for a public good may be revealed through majority voting, provided that individual preferences are single peaked. But even though the preferences are single peaked, it does not follow that the result from the majority voting process is optimal in the Pareto sense. It may be shown, however, that if the preferences are distributed among consumers in such a way that the me-

dian equals the mean, the voting outcome implies that the total demand price for a public good ($Q$) is equal to the marginal cost of supplying the public good. In order to show this, recall that individual i's demand price for the public good $Y$ is given by

$$Q_i = -\frac{\partial E_i}{\partial Y}.$$

Assume that each consumer has to pay a fixed share $a_i$ of the total cost of supplying this public good, where $\sum_{i=1}^{m} a_i = 1$. Then the optimal behavior of individual $i$ is determined by maximization of his consumer surplus less his share of the total cost of producing the public good:

$$I_i - E_i(P, Y, \bar{U}) - a_i C(Y)$$

where $C(Y)$ is the total cost function. Therefore, a necessary condition for maximization is

$$-\frac{\partial E_i}{\partial Y} - a_i MC = 0$$

or

$$Q_i = a_i MC$$

where $MC$ represents the marginal cost of producing the public good.

If we now assume that for $Y$, $Q_i$ is distributed over $m$ individuals in such a way that the mean ($\sum Q_i/m$) is equal to the median, it follows that the outcome of the majority voting will be a supply of $Y$ such that

$$Q = \sum Q_i = \sum a_i MC = MC.$$

This outcome will be the same as when the median voter's demand price ($Q_d$) is equal to his marginal cost share ($MC_d$). This can be proved as follows, using Bowen's crucial assumption that the mean equals the median. In the above, we have

$$\sum Q_i = \sum a_i MC$$

$$\sum a_i = 1.$$

Dividing both sides by $m$ yields

$$\frac{\sum Q_i}{m} = \frac{\sum a_i MC}{m}$$

which is

$$\frac{\sum Q_i}{m} = \frac{MC}{m}.$$

Since by definition

$$\frac{\sum Q_i}{m} = Q_d \quad \text{and} \quad \frac{MC}{m} = MC_d,$$

we have

$$Q_d = MC_d.$$

This is Bowen's result. His crucial assumption is that consumer preferences are distributed symmetrically about the median.

2. Now let us again consider Bowen's model expressed in terms of individual i's consumer surplus less his share of the total cost of supplying $Y$:

$$\begin{aligned} & I_i - E_i(P, Y, \bar{U}) - a_i C(Y) \\ = \; & I_i - E_i(P, Y, \bar{U}) - a_i MC \cdot Y \\ = \; & I_i - E_i(P, Y, \bar{U}) - a_i(\textstyle\sum MC_i/m) Y \\ = \; & I_i - E_i(P, Y, \bar{U}) - (\textstyle\sum Q_i/m) Y \\ = \; & I_i - E_i(P, Y, \bar{U}) - Q_d Y. \end{aligned}$$

The last term, $Q_d Y$, implies that individual *i* will have to pay $Q_d Y$. Therefore, $Q_d$ is his true marginal willingness to pay for $Y$. Keeping this in mind, consumers are asked how much they are willing to pay for a unit increase in the supply of $Y$. Consumers have a strong incentive to conceal their true preferences and hence their true marginal willingness to pay, in order to obtain benefits at the expense of other consumers.

Assume that each consumer *thinks* that he knows the amount all other consumers together are willing to pay for $Y$ and the quantity of $Y$ supplied by the community. Let his expected supply function for the public good $Y$ be

$$Y^e = Y^e(\bar{Q} + Q_i)$$

where $Y^e$ is the quantity of $Y$ consumer *i* expects to be supplied and $\bar{Q}$ is the total "stated" amount consumer *i thinks* all other consumers together are willing to pay for $Y$. Then the consumer's optimal behavior can be characterized by maximizing the difference between his consumer surplus and his cost:

$$I_i - E_i(P, Y^e, \bar{U}) - Q_i Y^e$$

subject to

$$Y^e = Y^e(\bar{Q} + Q_i).$$

The first-order conditions for this problem are

$$-\frac{\partial E_i}{\partial Y^e} - Q_i - \lambda = 0$$

$$-\Upsilon^e+\lambda\frac{d\Upsilon}{dQ}=0$$

where $\lambda$ is a Lagrange multiplier. These conditions imply that

$$(4)\quad Q_i=-\frac{\partial E_i}{\partial \Upsilon^e}-\Upsilon^e\frac{dQ}{d\Upsilon}$$

where $(-\partial E_i/\partial \Upsilon^e)$ is the "true" demand price for the public good $\Upsilon$ and $Q_i$ is the "stated" demand price for the public good $\Upsilon$. Since $dQ/d\Upsilon > 0$, it is thus optimal for the consumer to understate his demand price for public good. If consumers have perfect information on the supply function of $\Upsilon$, the authority may calculate this bias by estimating $\Upsilon^e dQ/d\Upsilon$ for each consumer. By correcting their answers for this bias, the authority may obtain the true demand price of $\Upsilon$.

But consumers usually differ in their beliefs about how much all other consumers are willing to pay and of the amount of $\Upsilon$ produced. To overcome this difficulty, one may introduce a density function for the probability distribution of the amount all other consumers are willing to pay, $f(\bar{Q})$, and incorporate it into the above analysis. This task is far from easy and nobody has yet clarified the scope of the problem in this way.

3. At this moment one may notice that the analysis presented in section IV-2 is similar to the analysis of monopolistic competition in market theory. This similarity highlights the point that the number of consumers demanding public goods in the community is sufficiently large that the actions of one individual have no perceptible influence on other consumers. This point may lead one to the following hypothetical approach to derive the true demand function for a public good.

Let us consider Figure 1, where we have three hypothetical individual

FIGURE 1

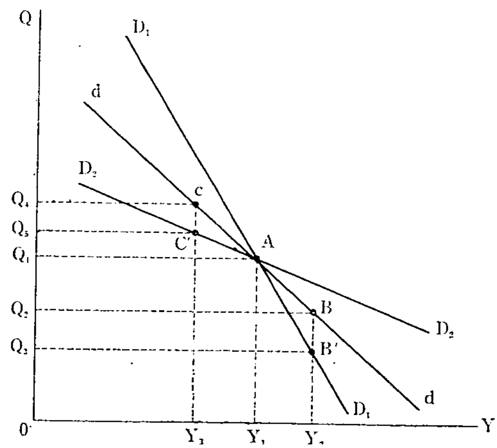
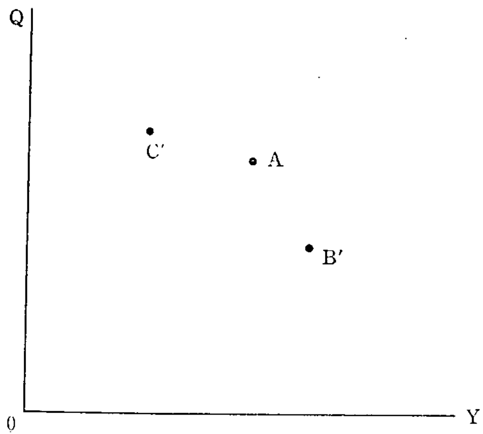


FIGURE 2




demand curves for the public good $Y$. One is the "true" demand curve and the other two the "stated" demand curves. Assume that the true demand curve (dd) is more elastic than one stated curve ($D_1D_1$) and less elastic than the other stated curve ($D_2D_2$).

Suppose that individual $i$ is at point A, where he pays $Q_1$ for a unit of $Y$, and the supply of $Y$ is $Y_1$. Suppose further that $Q_1$ is his ture demand price. Now, he is asked how much he is willing to pay for $Y$ as the supply of $Y$ increases a unit from $Y_1$ to $Y_2$. Then we can be sure that he is willing to pay $Q_3$ (the stated price) instead of $Q_2$ (the true price), so that he will stay at $B'$. The difference between $Q_2$ and $Q_3$ may be interpreted as his bias measured by $Y^e dQ/dY$ in (4). By the same token, he is again asked how much he is willing to pay for the supply of $Y$ as it decreases a unit from $Y_1$ to $Y_3$. Obviously, he is willing to pay $Q_5$ instead of $Q_4$, so that he will stay at $C'$. From this observation, one can expect that this consumer moves along the stated demand curve $D_1D_1$ when the supply of $Y$ increases, and that he moves along the stated demand curve $D_2D_2$ when the supply of $Y$ decreases. As a result, one can obtain three different points, shown in Figure 2.

Assume that his bias is symmetrical in a sense that his bias associated with a unit increase of $Y$ is equal to his bias associated with a unit decrease of $Y$. That is, $CC' = BB'$ in Figure 1. With this assumption, one can derive the true demand curve of the public good $Y$ which is parallel to $B'C'$, passing through point $A$ in Figure 2. But the question on which this thesis falls is where the three demand curves interesct. This remains unsolved.

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