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A STUDY OF STABILITY CONDITIONS FOR THE IMPERFECT WORLD WHEAT MARKET: A DUOPOLY SIMULATION MODEL*

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I. Introduction

1. *Objective*

It is the purpose of this study to develop and test a dynamic econometric model to be used to analyze the world wheat market and to identify the important behavioral and structural variables likely to affect future wheat market flow and prices under imperfect market conditions. Specifically this study is conducted to establish stable conditions for the world wheat market under imperfect competition.

2. *Justification*

Increased attention to results from economic models clearly point out the need for serious reexamination of certain traditional theories. Among these are theories in the area of unbalanced food grain demand and supply schedules relative to imperfect market competition in world wheat transactions. Development of a realistic economic model to be used in dealing with the problems of future market stability for an international commodity like wheat deserves a high priority. Thus the construction of a meaningful theory upon which to base policy and programs leading to stable trade flows under imperfect market conditions is urgently needed in order to avoid unnecessary price competition and to establish a more stable marketing system.

3. *Wheat Market Model*

A wheat market model to be used to measure the responsiveness to reaction behavior in duopolistic competition in the world wheat market has been developed. The purpose of this model is to obtain a solution for stable market conditions given optimum price reaction behavior within

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the stability conditions. And as a result of obtaining optimal conditions for stability, a behavioral adjustment process will be investigated to estimate stable prices for duopolists under these optimal conditions.

4. *Estimation Methods*

A generalized least squares estimation method used to estimate parameters in linear functions is introduced in this study. Even though highly sophiscated estimation techniques are applied to linear regression models, in any econometric model using time series data it is generally known that the removal of autoregression was not accomplished until this method was adopted. To obtain precise and efficient estimates is our immediate concern.

Within the class of linear unbiased estimates of the parameters, it is the generalized least squares estimates that minimize the variance of each estimate. The generalized least squares estimates have certain desirable properties. They are:

1) unbiased,
2) maximum likelihood estimates if normality is assumed,
3) the best linear unbiased estimates even if normality is not assumed, and
4) generalized least squares estimates with the smallest variance compared to other estimates in the case of hetroscedasticity existence, if the covariance structures are known.

5. *Literature Review*

Schemitz and Bawden[1] approached empirical analysis of the world wheat sector by using a spatial equilibrium analysis of the world wheat trade. General and broad trade theories have been applied to the solution of wheat transactions, however, they completely ignored the mechanism of market sectors.

Thus far, not very many works have dealt with the world wheat market under imperfect competition. There are, however, several contributions to this literature.

McCalla[2] approached world wheat trading strategy in terms of domestic policies in the international wheat market through the construction of imperfect competition theory. Geer[3] argues that price movement in the world coffee market under imperfect competition approximates

[1] A. Schmitz and L. L. Bawden, *World Wheat Economy*, California Agr. Exp. Stat., No. 32, March 1973, UC Berkeley.

[2] A. C. McCalla, *A Duopoly Model of World Wheat Pricing*, Minnesota Agr. Exp. Stat. Misc, J-Ser. No. 1353.

[3] T. Geer, "Price Formation on the World Coffee Market and Its Implications for the International Coffee Agreement," *Weltwirt Schaflicher Archiv*, 1971 106(1) pp. 128–152.

oligopoly.

Ferguson and Pfouts[4] presented a dynamic solution of duopolistic price competition in terms of price and game theory.

Among others, Mendelsohn[5] contributed a theoretical formulation to static equilibrium solutions for non-pure competition in international trade.

Besides those mentioned above, Farnsworth's[6] argument for an international wheat agreement to stabilize the world wheat price in terms of two pricing system concepts was carefully reviewed.

II. World Wheat Market

1. *Characteristics of the World Wheat Market*

The wheat market model in this study attempts to explain past wheat price movements and serves as a framework for analyzing possible future wheat price trends under imperfect competition in the world wheat market.

The conditions of international trade in wheat are greatly influenced by inward-looking domestic agricultural policies, whereas the international market is inherently unstable. Thus, future behavior must be analyzed in the context of an imperfectly competitive framework which makes adequate allowance for non-economic forces and domestic policy considerations to enter the analysis as parameters and behavioral functions. However, these non-economic factors will not be included in the model building.

From an institutional and statistical viewpoint, the wheat market may be described in terms of trade flows and announced prices but little inputation is made regarding the underlying price mechanism.[7] There are, however, three additional references to the imperfectly competitive nature of the world wheat market in terms of the price mechanism.

First, Mendelshohn argues that to consider international pricing in the framework of classical competitive trade theory is inappropriate and that the world price of wheat is directly affected by monopolistic and monopsonistic forces exercised through the International Wheat Agreement.[8]

[4] C. Ferguson, and R. Pfouts, "Learning and Expectations in Dynamic Duopoly Behavior," *Journal of Behavioral Science*, No. 20, 1960, pp. 225–236.

[5] C. Mendelsohn, "Approaches Formulation and Static Equilibrium Solutions of Non-pure Competition for International Trade," *Journal of Farm Economics*, No. 39, (December 1957), pp. 1726–31.

[6] H. C. Farnsworth, "International Wheat Agreements and Problems," *Quat. J. Econ*, No. 66, (May 1956), pp. 217–248.

[7] Malenhaum, W. W., *The World Wheat Market*, Cambridge, Harvard, 1953, pp. 152–153.

[8] Mendelsohn, C. *op. cit.*

Second, Farnsworth has identified the oligopolistic nature of world wheat pricing but has not explicitly outlined a plausible oligopolistic model.[9]

Finally, Holland and Thomas have argued intuitively that Canada and the U.S. act as duopolists.[10] A cooperative duopoly model attempts to explore competitive conditions which are said to exist if a country is either unable or unwilling to exert market power, in other words, the ability and willingness of an economic unit to influence price.[11]

2. *Duopoly Market Model*

The market model in this study suggest assumptions which are *a priori* reasonable and yield stable solutions for duopolistic market competition as well. The basic assumptions for a solution for duopolistic competition in the world wheat market are justified by the following:

1) Wheat will be assumed to be a homogenous product in terms of a marketable commodity. While quality differences among various classes of wheat are recognized, it is believed that U.S. Hard Red Winter wheat is sufficiently representative of several varieties of wheat including the Canadian that the assumption of homogeneity is realistic.
2) The U.S. and Canada have supplied about 69 percent of wheat demanded in the world market during the past two and a half decades, and only these two countries have storage facilities in sufficient volume to permit stockpiling. Therefore, these two countries can be used to illustrate the duopolistic model.

The behavioral interdependence of duopolists is illustrated schematically in Figure 1 and Figure 2.

Figure 1 illustrates the impact channels of factors influencing the duopoly decision making process as indicated by arrow direction and Figure 2 illustrates the functional relationships to be used in building the duopoly market model.

The main concern is to develop a working model consistent with the essential properties of a duopoly market. The purpose of a market model for duopolists is to obtain a solution for stable market conditions at an optimum price reaction behavior within these stability conditions.

The necessary assumptions identical with reality in terms of economic behavior are:

[9] Farnsworth, H.C. "International Wheat Agreements and Problems, 1949–1956," *Quarterly Journal of Economics*, Vol. 66, No. 2, May 1956, pp. 217–220.

[10] Holland, H.F. and Thomas, M.D. "The World Price," Special Oregon State Wheat Paper, Cooperative Extension Service, Oregon State University, December 1964.

[11] McChalla, A.F., "A Duopoly Model of World Wheat Pricing," *Journal of Farm Economics*, Vol. 48, No. 3, Aug. 1966, p. 712.

FIGURE 1 Impact Paths of the Decision-Making Process

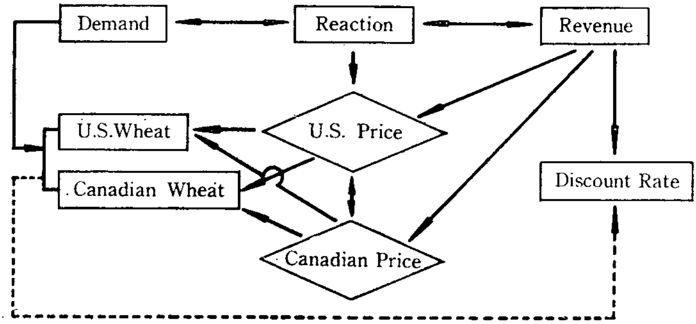
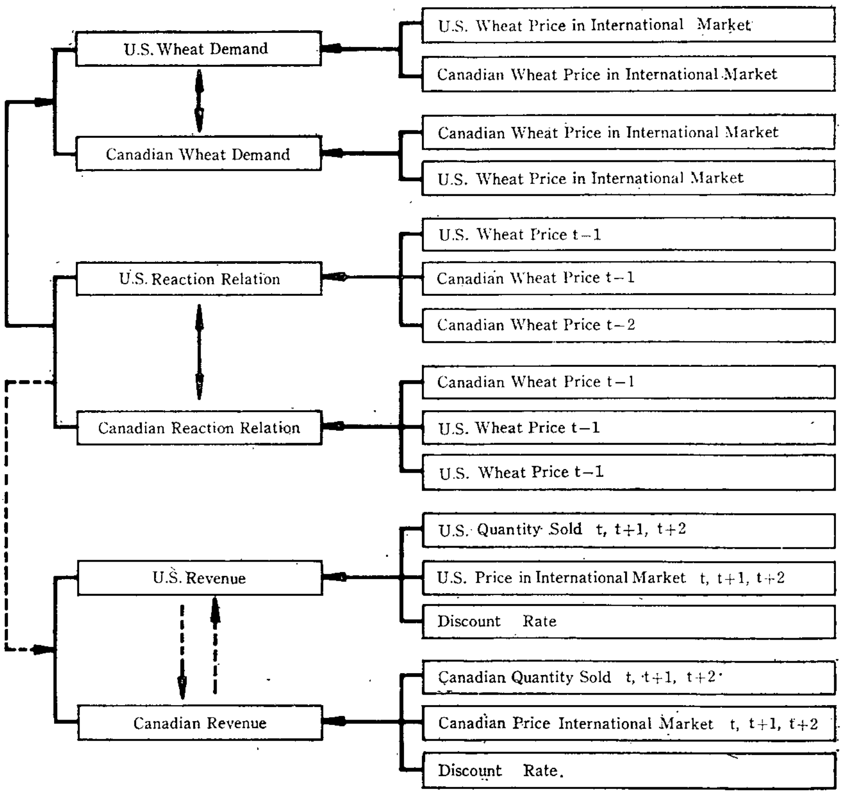


FIGURE 2 Schematic Diagram of a Duoploy Market Model



1) The market model presented here accounts only for the behavior of Competitors who expect rivials to influence and be influenced by their matual interdependence.
2) The duopoly model is dynamic in the sense that the model involves time movement between periods with the various behavioral hypotheses regarded as fixed and constant over any

given time period.

3) Using prices as parameters of action, the information feedback will be utilized in measuring the conditions of duopolistic price competition, and
4) Linear price reaction and market demand functions are assumed to avoid a system of high order, nonlinear relations which are generally considered unsolvable.

The main objective of utilizing a market model is to find the necessary and sufficient conditions for duopolists and, consequently, determine stable price levels. As a result of obtaining stable conditions, quantity adjustment with respect to maximizing sales revenue will be investigated.

III. Building the Model

Science is not the collection of miscellaneous facts, it is rather the orderly classification of facts and an explanation of their interrelationships. The so-called pure theory of economics is based upon logical reasoning as to how men would act under certain stated conditions. Such pure abstract reasoning can be an end in itself just as it was in the geometry of the ancient Greeks.[12]

While some are interested mainly in economic theory, others want to forecast expected economic phenomena such as prices, or at least to estimate the prices that would be expected under certain assumed conditions. For this purpose, any economic theory must be concrete, quantitative and statistical. Such work calls for applied economic theory. But sometimes the main emphasis has been upon measurement, and theory has been kept in the background. But just as good theory is essential to good forecasting, good statistical measurement and statistical analysis are essential to developing economic theories that adequately describe how economic variables actually react in real economic phenomena.

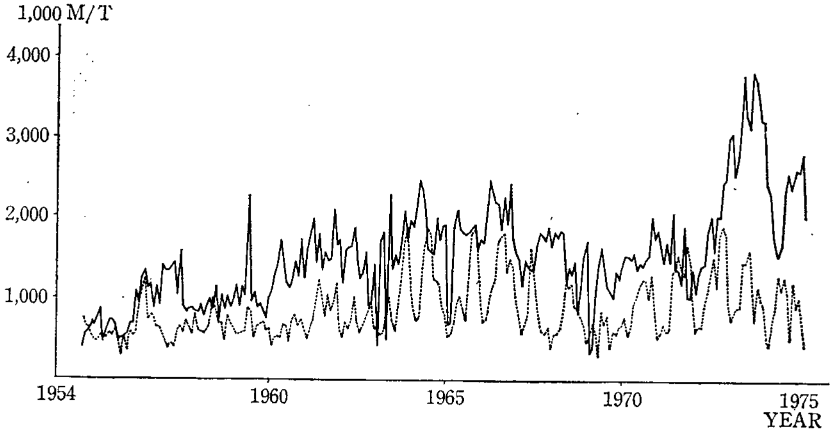
There are several reasons to identify the model with reality. Canada has been the largest commercial wheat exporter since the early 1940's and Canada's premium quality allowed Canada to be a barometric price leader in the world wheat market competition between the U.S. and Canada.

The U.S. wheat policy, especially prior to July 1964, of guaranteeing a support level above world price necessitates the payment of export subsidies for commercial export. Thus, the setting of an export subsidy would determine the world price.

Given that the Canadian Wheat Board has direct control over export price as opposed to the indirect U.S. control, the U.S. position is simplified if it can take the Canadian price as a referent and determine the export subsidy. Thus, the Canadian Wheat Board has a policy of maximizing total sales given the constraints of the initial payment plus the Board operating costs and the supply situation prevailing in Canada.

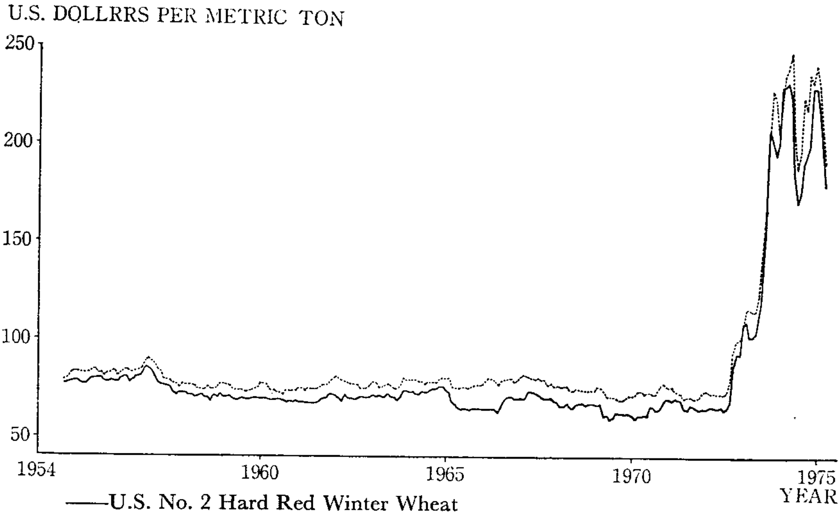
[12] H.L. Moore, *Synthetic Economics*, MacMillan, New York, 1929, p. 8.

FIGURE 3 International Wheat Exports, 1955–75



——U.S. Wheat Exports (No. 2 HRW: Hard Red Uinter Wheat)
······Canadian Wheat Exports (No. 1 WRS: Western Red Spring Wheat)

FIGURE 4 International Wheat Price: CIF Rotterdam, 1955–75



——U.S. No. 2 Hard Red Winter Wheat
······Canadian No. 1 Western Red Spring Wheat

The strong competitive positions in the world wheat market of the U.S. and Canada are represented in the following figures in terms of monthly volume reports and Rotterdam market prices. These two figures show how these two countries have been competing in the wheat export market historically.

1. *Mathematical Model*

The model in this study accounts only for the behavior of duopolists who expect each other to influence and be influenced by the interdependence among duopolists in the world wheat market. A behavioral hypothesis explicitly requires that each duopolist anticipates competitive reactions by his rival. The hypothesis must express the manner in which the U.S. duopolist expects the Canadian duopolist to react to changes in the action variables usually called "parameters of action."[13]

The model to which we are referring is dynamic in the sense that the entire model involves time in an essential way. The various behavioral hypotheses employed are regarded as fixed and constant.

Since the cost functions for each duopulist are not known, demand and reaction functions are assumed to build the simplest possible model consistent with the essential properties of duopoly markets. All parameters are linear relations. The assumptions of linearity will be adequate to describe the duopoly model.[14]

A. *Demand Equations:*

(1) $$Qus_t = \alpha_0 - \alpha_1 Pus_t + \alpha_2 Pcan_t$$
$$Qcan_t = \beta_0 - \beta_1 Pcan_t + \beta_2 Pus_t$$

B. *Reaction Equations:*

(2) $$Pus_t = Pus_{t-1} + a_1(Pcan_{t-1} - Pcan_{t-2})$$
$$Pcan_t = Pcan_{t-1} + b_1(Pus_{t-1} - Pus_{t-2})$$

C. *Revenue Equations:*

(3) $$Rus_T = Pus_t\ Qus_t + r_1\ Pus_{t+1}\ Qus_{t+1} + r_2\ Pus_{t+2}\ Qus_{t+2}$$
$$Rcan_T = Pcan_t\ Qcan_t + r_1\ Pcan_{t+1}\ Qcan_{t+1} + r_2\ Pcan_{t+2}\ Qcan_{t+2}$$

where $Qust_t$ = U.S. No. 2 HRW exported at time t (1,000 m.t.)
$Qcan_t$ = Canadian CWRS No. 1 exported at time t (1,000 m.t.)
Pus_t = Rotterdam price (c.i.f.) of U.S. No. 2 HRW at time t (U.S. $/100 kg)
$Pcan_t$ = Rotterdam price (c.i.f.) of Canadian CWRS No. 1 at time t (U.S. $/100 kg)
Rus_T = Sales Revenue for U.S. over 3 time periods
$Rcan_T$ = Sales Revenus for Canada over 3 time periods, where T = t, t+1, and t+2
r = discount rate.

Nominal Short-term Prime rate:
1975 average prime rate r = 0.78625%.
Source: Federal Reserve Bulletin, Vol. 62, No. 3, March 1976, Table A 62.,

where $r_1 = \dfrac{1}{(1+r)}$

[13] Prices considered as parameters of action.

[14] Samuelson, P.A., *Foundations of Economic Analysis* (Cambridge: Harvard University Press, 1947), Ch. 10, p. 108.

$$r_2 = \frac{1}{(1 + r)^2}$$

The demand equation, *per se*, is assumed to be known with certainty, but the volume of sales expected for a certain period is not known since it depends on the price actually charged by the rival duopolist. According to the assumptions previously made, the U.S. and Canada choose their price Pus_t and Pcan_t respectively, so as to maximize their revenues. However, since reaction equations involve lagged values of the price variables, each duopolist believes that the price he establishes in period t will affect his rival's price and, consequently, his own revenue in time periods $t + 1$ and $t + 2$.

Therefore, in determining the optimal price at time t, the duopolists actually maximize over a three-period time span and the revenues of periods $t + 1$ and $t + 2$ are discounted by r_1 and r_2 respectively.

2. *Stability Conditions*

The nature of the reaction equations under consideration is directly related to a learning process in a duopoly game, that is periodic adjustment of the response pattern as knowledge is accumulated. However, an admissible rule has two essential properties. First, it will enable a competitor eventually to forecast his rival's equilibrium price. Second, once the equilibrium is attained, the rule will continue to predict this same equilibrium price in each succeeding period.

The admissible rules we are referring to may be characterized as follows:

$$\begin{aligned}
(4) \qquad & \Delta\text{Rus} > 0 \text{ for } \text{Pus}_t - \text{Pus}_{t-1} \neq 0 \\
& \Delta\text{Rcan} > 0 \text{ for } \text{Pcan}_t - \text{Pcan}_{t-1} \neq 0 \\
& |\text{Pus}_t - \bar{\text{p}}\text{us}_{t+n}| < |\text{Pus}_{t-1} - \bar{\text{p}}\text{us}_{t+n}| \\
& |\text{Pcan}_t - \bar{\text{p}}\text{can}_{t+n}| < |\text{Pcan}_{t-1} - \bar{\text{p}}\text{can}_{t+n}|
\end{aligned}$$

Where $\bar{\text{p}}$ indicates equilibrium prices.

The equilibrium prices have been realized and continued application of the rules for altering the reaction function yields the prediction that equilibrium prices will prevail. Consider a set of rules the elements of which have the following property: in period $t + 1$ a duopolist changes his reaction function in such a way that the altered function would have predicted the competitor's price for period $t + 1$ correctly if it had been used in period t. For example, U.S. (Canada) changes the intercept of its linear reaction equation in period $t + 1$ to that value which would have given the correct price of Canada (U.S.) if it had been used in period t.

To develop the use of this rule in a linear model we consider Canada at time period $t+1$ and estimate the U.S. price by using the following linear equation:

$$\text{Pus}_{t+1} = b_0 + b_1 \text{ Pcan}_t$$

That is, the U S. price in the $t+1$ period is a linear function of the Canadian price in period t. Thus, Canada can find the value of intercept b_0 which would give a correct prediction:

$$b_0 = \text{Pus}_{t+1} - b_1 \text{ Pcan}_t$$

Now, suppose that Canada uses the expected value of $\hat{a}_0$ as its intercept. In the equation to predict Pus_{t+2}, simply set $b_0 = b_0^*$; thus:

$$\begin{aligned}\text{Pus}_{t+2} &= b_0^* + b_1 \text{ Pcan}_{t+1}\\ &= \text{Pus}_{t+1} - \text{b}_1 \text{ Pcan}_t + b_1 \text{ Pcan}_{t+1}\end{aligned}$$

Therefore, $\text{Pus}_{t+2} = \text{Pus}_{t+1} + \text{b}_1 (\text{Pcan}_{t+1} - \text{Pcan}_t)$

By extending this equation to the general system of reaction equations we have the following:

$$\begin{aligned}\text{Pus}_t &= \text{Pus}_{t-1} + \text{b}_1 (\text{Pcan}_{t-1} - \text{Pcan}_{t-2})\\ \text{Pcan}_t &= \text{Pcan}_{t-1} + a_1 (\text{Pus}_{t-1} - \text{Pus}_{t-2})\end{aligned}$$

If equilibrium is reached, $\text{Pcan}_{t-1} = \text{Pcan}_{t-2}$ and $\text{Pus}_{t-1} = \text{Pus}_{t-2}$ and stability is generally contingent upon two factors: learning and expectation. Learning is the element which makes it possible to change the pattern of expected response. Using equations (1), (2) and (3) and condition set (4), we obtain the following set of linear difference equations.[15]

$$\begin{pmatrix} -2\alpha_1 & \alpha_2 \text{a}_1 & -\alpha_2 \text{a}_1 & 0 & \alpha_2 & 0 \\ \alpha_2 \text{a}_1 - \text{r}\alpha_1 & \alpha_2 \text{a}_1 \text{r} - 2\alpha_1 & \alpha_2 \text{a}_1 (1-r) & 0 & r\alpha_2 & \alpha_2 \\ 0 & \beta_2 & 0 & -2\beta_1 & \beta_2 \text{b}_1 & -\beta_2 \text{b}_1 \\ 0 & r\beta_2 & \beta_2 & \beta_2 b_1 - r\beta_2 & \beta_2 b_1 r - 2\beta_1 & \beta_2 b_1 (1-\text{r}) \\ 0 & 0 & r\beta_2 & r^2 \beta_2 b_1 & r\beta_2 b_1 - \beta_1 & r\beta_2 b_1 - 2\beta_1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Pus}_{t+2} \\ \text{Pus}_{t+1} \\ \text{Pus}_t \\ \text{Pcan}_{t+2} \\ \text{Pcan}_{t+1} \\ \text{Pcan}_t \end{pmatrix} = \begin{pmatrix} -\alpha_0 \\ \alpha_2 \text{a}_1 \text{Pus}_{t-1} - \alpha_0 \\ \alpha_2 a_1 (r-1) \text{Pus}_{t-1} + \alpha_2 a_1 \text{Pus}_{t-2} - \alpha_2 \text{Pcan}_{t-1} - \alpha_0 (1+r) \\ -\beta_0 \\ \beta_2 b_1 \text{Pcan}_{t-1} - \beta_0 \\ \beta_2 b_1 (r-1) \text{Pcan}_{t-1} + \beta_2 b_1 \text{Pcan}_{t-2} - \beta_2 \text{Pus}_{t-1} - \beta_0 (1+r) \end{pmatrix}$$

From the above structural system, a linear difference equation system was derived.

(5) $(\text{I} - A)P = K$

where $\text{I} = 6 \times 6$ idendity matrix

$\text{A} = 6 \times 6$ metrix

[15] Solon, R., "On the Structure of Linear Models," *Econometrica*, 1952, Vol. 50, pp. 29–46.

Chipman, J.S., "The Multisector Multiplier," *Econometrica*, 1950, vol. 48, pp. 355–374.

D = 6 × 1 column vector
K = 6 × 1 column vector.

$$A = \begin{pmatrix} 0 & \frac{\alpha_2 a_1}{2\alpha_1} & -\frac{\alpha_2 a_1}{2\alpha_1} & 0 & \frac{\alpha_2}{2\alpha_1} & 0 \\ \frac{\alpha_2 a_1}{2\alpha_1}-\frac{r}{2} & \frac{\alpha_2 a_1 r}{2\alpha_1} & \frac{\alpha_2 a_1}{2\alpha_1}(1-r) & 0 & \frac{r\alpha_2}{2\alpha_1} & \frac{\alpha_2}{2\alpha_1} \\ \frac{r^2\alpha_2 a_1}{2\alpha_1} & \frac{r\alpha_2 a_1}{2\alpha_1}-\frac{1}{2} & \frac{r\alpha_2 a_1}{2\alpha_1} & 0 & 0 & \frac{r\alpha_2}{2\alpha_1} \\ 0 & \frac{\beta_2}{2\beta_1} & 0 & 0 & \frac{\beta_2 b_1}{2\beta_1} & -\frac{\beta_2 b_1}{2\beta_1} \\ 0 & \frac{r\beta_2}{2\beta_1} & \frac{\beta_1}{2\beta_1} & \frac{\beta_2 b_1}{2\beta_1}-\frac{r}{2} & \frac{\beta_2 b_1 r}{2\beta_1} & \frac{\beta_2 b_1}{2\beta_1}(1-r) \\ 0 & 0 & \frac{r\beta_2}{2\beta_1} & \frac{r^2\beta_2 b_1}{2\beta_1} & \frac{r\beta_2 b_1}{2\beta_1}-\frac{1}{2} & \frac{r\beta_2 b_1}{2\beta_1} \end{pmatrix}$$

A stability system such as (5) depends on the moduli of the characteristic roots of A.

Necessary and sufficient conditions for the stability of the system which may be stated in terms of the elements of A are also known.

Coefficients a_1 and b_1 indicate the sensitivity of the U.S.'s and Canada's expectations; they indicate whether expectations are "adaptive" or "extrapolative." One should not expect negative values for a_1 and b_1 when price is treated as the parameter of action, although the expectation coefficients are frequently negative in models which use quantity as the opposite parameter.

However, for the necessary and sufficient condition for stability, the determinant of matrix A should lie between zero and one.

For the time being, the main concern is to find the values of reaction coefficients a_1 and b_1 which satisfy the condition that the determinant of matrix A is less than one but greater than zero.

IV. Solution of the Model

Contributions to the dynamic theory of the duopoly market model have not been numerous. Evans and Ross[16] were the early pioneers in this field, using the calculus of variations to determine the optimal time path of the price function. More recently, Pfouts[17] has used pure difference equations in his approach to the dynamic theory of interrelated markets.

The results of these studies have generally shown that dynamic stability

[16] Evans, G.C., *Mathematical Introduction to Economics*, McGraw-Hill, New York, 1930, p. Ross, C. F., *Dynamic Economics*, Indiana Univ. Press, Bloomington, Ind., 1934.

[17] Pfouts, R.W. and Ferguson, C. E., "A Matric General Solution of Linear Difference Equations with Constant Coefficients," Mathematical Magazine, 1960, Vol. 33, pp. 119–127

is attainable only if sufficiently simple conjectural behavior patterns are postulated for each duopolist.

The purpose of formulating a simple duopoly market model for the world wheat trade flow is to present a set of assumptions which are *a priori* reasonable and which yield solutions for duopolistic market competition for the wheat trade. However, the central theme in this context is that the learning process and expectations are the factors which together determine stability in a dynamic duopoly market. As described above, the market model developed in this study accounts for the behavior of each duopolist who expects his rival to influence and be influenced by the mutual interdependence between duopolists.

A behavioral hypothesis is one which explicitly requires that each duopolist anticipates competitive reactions by his rival. The hypothesis must express the manner in which a duopolist expects his rival to react to changes in his action.

The model to which we are referring in this study is dynamic in the sense that it involves time concepts; however, the various behavioral hypotheses employed here are regarded as fixed and constant over time. The market demand function for duopolists is assumed to be known with certainty, however, the volume of exports anticipated for a certain period is not certain since it depends on the price actually charged by the rival duopolist.

Our main concern is to find the reaction coefficient values which meet the assumptions.

The following demand equations for duopolists are computed by the ordinary and generalized least squares estimation methods. The results are:

i) U.S.A.

OLS: $Qus_t = 759.7102 + 4.0759\, Pus_t + 1.1307\, Pcan_t$

$\qquad (13.6589) \qquad (12.2734)$

$R^2 = 0.54$, D.W. $= 0.713$, MSE $= 89044$

GLS: $Qus_t = 846.8937 - 13.577\, Pus_t - 21.0833\, Pcan_t$

$\qquad (14.0857) \qquad (13.9429)$

$R^2 = 0.50$, D.W. $= 1.612$, MSE $= 68448$

ii) Canada

OLS: $Qcan_t = 438.507 + 3.5856\, Pcan_t + 0.2457\, Pus_t$

$\qquad (6.3657) \qquad (7.0842)$

$R^2 = 0.632$, D.W. $= 1.051$, MSE $= 23953$

GLS: $Qcan_t = 427.283 - 0.3845\, Pcan_t + 4.1226\, Pus_t$

$\qquad (11.7038) \qquad (12.2428$

$R^2 = 0.57$, D.W. $= 1.312$, MSE $= 22165$

Our main purpose in choosing statistical models with better fit is to obtain better estimated parameters for reaction equations in the duopoly market model. The period used in the regression analysis is 1955–74. The figures in parenthesis denote the standard errors of the estimated parameters. With chosen reaction coefficients from GLS, matrix A is formulated to test the stability conditions for duopolistic market competition in the world wheat market. The chosen coefficients are: $\alpha_0 = 846.8937$, $\alpha_1 = -13.577$, $\alpha_2 = 21.0883$, $\beta_0 = 427.283$, $\beta_1 = -0.3845$ and $\beta_2 = 4.1226$. By manipulating matrix A we will find the values of a_1 and b_1 in the reaction equations to satisfy the stability conditions for duopolistic competition. The determinant of matrix A should be less than unity but greater than zero. By building a computer simulation program and applying this to the solution all levels of a_1 and b_1 values are generated.

For the purpose of computational procedure the following determinant of matrix A is formulated

$$(6) \qquad |A| = \left(\frac{1}{2\alpha_1}\right)^3 \left(\frac{1}{2\rho_1}\right)^3$$

$$\begin{pmatrix} 0 & \alpha_2 a_1 & -\alpha_2 a_1 & 0 & \alpha_2 & 0 \\ \alpha_2 a_1 - r_1 & \alpha_2 a_1 r_1 & \alpha_2 a_1 - r_1 \alpha_2 a_1 & 0 & r_1 \alpha_2 & \alpha_2 \\ r_2 \alpha_2 a_1 & \alpha_2 a_1 r_1 - \alpha_1 & r_1 \alpha_2 a_1 & 0 & 0 & -r_1 \alpha_2 \\ 0 & \beta_2 & 0 & 0 & \beta_2 b_1 & -\beta_2 b_1 \\ 0 & r_1 \beta_2 & \beta_2 & \beta_2 b_1 - r_1 \beta_1 & \beta_2 b_1 r_1 & \beta_2 b_1 - r_1 \\ 0 & 0 & r_1 \beta_2 & r_2 \beta_2 b_1 & r_1 \beta_2 b_1 - \beta_1 & r_1 b_1 \beta_2 \end{pmatrix}$$

By substituting elements of $|A|$ with a_{ij} elements, the following reduced $|A|$ is formulated ($i = 1, \ldots 6$ and $j = 1, \ldots 6$). The results of the computer simulation are presented in Table 1 first column and second column of Table 1 indicate reaction coefficient for the U.S. and Canada respectively.

From Table 1, Figure 5 was drawn. It illustrates the stability region which contains all the possible combinations of reaction coefficients for the U.S. and Canada which satisfy the condition $0 < |A| < 1$. The boundary of a stability region is always at the margin of change in reaction parameters to the adjustment of learning and expectations of duopolists, which justifies the hypothesis that each duopolist reacts continuously to adjust the parameters of action. Our next concern was focused upon an optimal condition of stability.

As shown in Figure 5, there are three optimal stability conditions developed as a result of this study. They are:

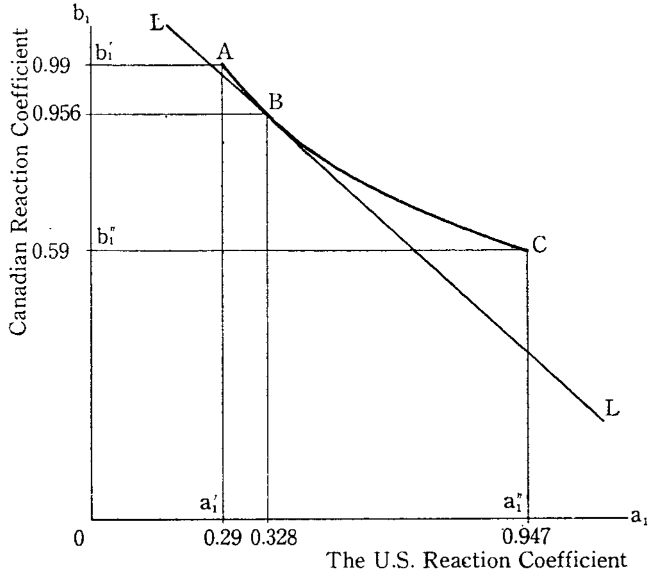
i) if $0 < a_1 < a_1'$ and $0 < b_1 < b_1'$, then optimal stability condition occurs at A,

ii) if $a_1' < a_1 < a_1''$ and $0 < b_1 < b''$, then optimal stability condition occurs at C, and

iii) if $a_1' < a_1 < a_1''$, and $b_1'' < b_1 < b_1'$, then optimal stability condition occurs at B.

In Figure 5, LL′ denotes a perfect duopoly reaction curve, which

FIGURE 5 Stability Region in Duoploy Market



illustrates a perfect competitive position.

Consider a straight line for LL′,

(7–1) $b_1 = f - a_1$

and suppose the AC curve is in a quadratic functional relation then

(7–2) $a_1 = \alpha + \beta b_1 + \gamma b_1^2 + e.$

Then optimality occurs at the point where the LL′ curve and the AC curve meet. This point is the unique point leading to an opitimal stability condition between A and C. Thus, point B is computed as follows: By taking the partial derivatives from equations (7–1) and (7–2), the slopes of these equations are obtained

From equation (7–1)

(7–3) $$\frac{\partial b_1}{\partial a_1} = -1.$$

And from equation (7–2)

(7–4) $$\frac{\partial b_1}{\partial a_1} = \beta + 2\gamma b_1.$$

Let the slope of the two curves be equal

(7–5) $$\left[\frac{\partial a_1}{\partial b_1}\right]_{\text{quadratic}} = \left[\frac{\partial a_1}{\partial b_1}\right]_{\text{linear}}$$

equations (7–3) to (7–5), yield an equation which can be solved for b

(7–6) $\beta + 2\gamma b_1 = -1.$

$$b_1 = \frac{-(\beta + 1)}{2\gamma}.$$

Substuting b_1 into (7–2), we can get the answer for a_1. By substuting each pairwise point on the AC curve Table 1 was established.

TABLE 1 REACTION COEFFICIENTS FOR STABILITY CONDITIONS IN CONTINOUS ADJUSTMENT BEHAVIOR

Reaction	Coefficients
a_1	b_1
0.29	0.99
0.30	0.98
0.31	0.97
0.32	0.96
0.33	0.95
0.35	0.94
0.36	0.93
0.38	0.92
0.40	0.90
0.45	0.85
0.50	0.82
0.55	0.79
0.60	0.76
0.65	0.73
0.71	0.70
0.75	0.68
0.79	0.66
0.81	0.65
0.85	0.63
0.90	0.61
0.94	0.60
0.947	0.59

How duoplists influence and are influenced in international wheat market competition by each other is discussed in this wheat economic model by utilizing Stackelberg's duopoly model which deals with cooperative behavioral patterns in market exploration. The results of this study have generally shown that dynamic stability is attainable only if sufficiently simple and continvous conjectural competitive patterns are postulated for each duopolist.

As stated in the objective of this study, an attempt to establish stability conditions for the duopolistic international wheat market was made. Optimal solutions for the U.S. and Canada were obtained. Table 2 indicates the optimal solutions for world wheat market competition in terms of reaction coefficients.

Optimal points in Figure 5 imply adjustment to changes in parameters of reaction with which the U.S. and Canada confront the occurance of

TABLE 2 OPTIMAL SOLUTION FOR STABILITY CONDITIONS

Range of Reaction Coefficients*	Optimal Reaction Coefficients	
	U.S.A.	Canada
$0 < a_1 < 0.29$ $0 < b_1 < 0.99$	0.29	0.99
$0 < a_1 < 0.947$ $0 < b_1 < 0.59$	0.497	0.59
$0.29 < a_1 < 0.497$ $0.59 < b_1 < 0.99$	0.328	0.9557

* a_1 and b_1 denote reaction coefficients for the U.S. and Canada respectively.

various conjectural hypotheses. In order to stabilize competitive market conditions in the world wheat market, the U.S. and Canada might agree to choose their reaction coefficients with respect to A, B, and C given the alternative decision-making procedures shown in Table 2. This also could avoid an unnecessary competitive motivation and as a result of this stabilize overall international wheat trade practices as far as these two competitors are concerned. In this study, however, only price was taken into consideration in the world wheat market model as a determinant factor of duopolistic competition. A detailed model including the impact of institutional factors on duopoly wheat market model building would result in a more realistic application. However, if the optimal stability conditions are conjectured in period $t + 1$ and so on, stable prices for dupolists corresponding to that period under the assumption of hypothetical optimality can be computed. The stable price here meant does not imply actual forecasted price, but indicates prices established through the market mechanism to avoid unnecessary price competition between duopolists. Table 3 illustrates the stable prices for the U.S. and Canda.

V. Conclusion

The international wheat trade has been subject to controls and re-

TABLE 3 STABLE PRICES FOR THE U.S. AND CANADA*

Optimal Stability Conditions	Stable Price	
	U.S.A.	Canada
		(Unit = US $/m.t.)
$a_1 = 0.29$ $b_1 = 0.97$	166.3920	169.6966
$a_1 = 0.947$ $b_1 = 0.59$	154.1406	185.3606
$a_1 = 0.328$ $b_1 = 0.9557$	165.1444	171.2226

* Rotterdam c.i.f. price.

strictions by many countries for a long period of time. Implementation of national agricultural trade policies by both exporting and importing countries have important effects on the allocation of wheat as well as pricing in the international wheat market. This study has attempted to address in part issues regarding future demand and supply schedules and imperfect market practices in the world wheat economy sector. Construction of meaningful models to be used in analyzing the wheat economy sector was one of the major objectives of this study.

Upon reviewing the wheat economic model with respect to future market situations wheat flows entering the international market may be more competitive. This situation may result in a more specific marketing strategy of securing market shares. An optimal strategy for this purpose was also attempted in this study. A two-country experimental case study was attempted assuming the presence of duopolistic competition in the international wheat market. The solution for duopoly market competition between the U.S. and Canada led to optimal solutions for stabilizing marketing and pricing in the international wheat trade. The stabilization of the international market directly affects domestic market stability.

In this study, however, an optimal solution to price competition between duopolists was obtained and optimal stable prices computed. Derived optimal prices under stable conditions are refered to as desirable prices for duopolists to use in avoiding unnecessary price competition. The optimal stability ranges obtained in this study by the computer simulation technique may be useful for duopolists in determining decision-making criteria under the above mentioned hypothetical conjectures as to the rival's behavior. Assuming any of the hypothetical occurences, the results of this study may be useful in the adjustment of a rival's behavioral reactions for the purpose of marketing stabilization.

Throughout this study, however, some new methods were attempted in analyzing the world wheat economic sector with respect to demand and supply schedules and market stabilization in the international wheat trade. Only partial solutions were obtainable due to limitations of data availability and assumptions necessary for simplicity.

A more detailed and realistic model without the limitations made in this study certainly deserves a high priority and consideration. As a contribution to achieving this goal, data listings and sources for the wheat sector have been compiled and prepared for future reference as a part of this study.

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