87

A LINEAR GOAL PROGRAMMING MODEL FOR FARM PLANNING OF SEMI-SUBSISTENCE FARMS

KANG BONG-SOON*

1. Introduction

Traditional single-objective approaches for farm planning assume that farmers aim to maximize total gross margin or profit from the resources at their command. However, even if the maximization of total gross margin or profit may be regarded as an important objective of farming, it is by no means the only one, nor is it necessarily the most pressing. This is particularly the case on semi-subsistence farms where there are many other objectives present, both financial and non-financial, arising largely from the economical, social and traditional characteristics of the farming society.

Farm business considerations of semi-subsistence farms are strongly connected with family considerations because a disastrous error in farming tends to be disastrous to the participating family as well. This may be expressed more directly as a desire for stable, rather than maximum, total profit or for the assurance that total gross margin is unlikely to fall below some specified level. And farmers do not generally like to borrow money from others unless it is absolutely necessary for farming. Moreover, most semi-subsistence farmers are overworked during the peak seasons due to the low level of mechanization and the high degree of seasonality in the demand for labor and, consequently they may desire to curtail their labor in this period. Under such multiple criteria for farming, the farm planning problem cannot be satisfactorily handled with conventional single-objective models if the multiple criteria conflict with each other.

Even though many approaches for dealing with the multiple objective problems have been developed and applied to various fields [19], most of them are difficult in their general application to farm planning

because of computational difficulties and additional information require-
ments which can not be easily met. Among those, an approach which
manages to overcome the difficulties is Linear Goal Programming (LGP)
suggested by Charnes and Cooper [8] and developed by Ijiri [22], Lee
[25] and Ignizio [20]. In LGP models, multiple objectives are explicitly
incorporated within an LP framework based on lexicographic ordering,
where the decision maker is assumed not to be prepared to allow trade-offs
between attributes (i.e., pre-emptive priorities are assumed). But the pre-
emptive priority assumption seems too restrictive.

This study mainly aims to develop a LGP model so as to reflect the
farming concerns of farmers more realistically and thus to be generally
applied to farm planning of semi-subsistence farms. But in constructing
the model special emphasis was placed on deriving a simpler method that
may be applied to farm planning handily and efficiently

II. Model Construction

1. General Linear Goal Programming Model

Faced with a multiple objective decision making problem, traditional
single objective approaches give priority to one of the objectives at the
expense of others which, due to the limitations of the methods, are essen-
tially ignored. In contrast, multiple objective approaches attempt to in-
clude all pertinent objectives. However, not all objectives may be opti-
mized. As a result, aspired levels of achievements or goals may be esta-
blished for each of these objectives. It may not be possible to satisfy even
the specified goals within the given constraints, but it is highly unlikely
that all of these objective goals are truly absolute. Consequently, for the
nonabsolute goals, one may have to be satisfied with the solution that comes
as close as possible to the satisfaction of all of these goals. That is, with
the solution that minimizes the deviation from the specified goals rather
than satisfies them absolutely. In essence, a goal programming approach
is based on this point.

Even though the notation used by those involved in goal programming
is by no means standardized, the general LGP model can be mathemati-
cally formulated as follows:

Find $x = (x_1, x_2, \ldots \ldots, x_n)$ so as to minimize

(2 — 1) $\underline{a} = \{P_1 r_1(\underline{d}^-, \underline{d}^+), P_2 r_2(\underline{d}^-, \underline{d}^+), \ldots \ldots, P_q r_q(\underline{d}^-, {}^+\underline{d})\}$

subject to

$$\sum_{j=1}^{n} g_{kj} x_j + d_k^- - d_k^+ = G_k, k = 1, 2, \ldots, z$$

$$\sum_{j=1}^{n} b_{ij} x_j + d_i^- - d_i^+ = B_i, i = 1, 2, \ldots, m$$

$x, d^-, d^+ > 0$

where $x =$ n dimensional decision variable vector

$a =$ q dimensional deviation function vector whose dimension represents the number of pre-emptive priority levels among the objectives[1]

$P_q =$ Pre-emptive priority associated with $r_q(\underline{d}^-, \underline{d}^+)$

$r_q(\underline{d}, \underline{d}^+) =$ linear function of the goal deviation variables, i.e., achievement function of the goal with qth ranked priority.

$g_{kj} =$ the coefficient of the jth decision variable for the kth goal constraint

$b_{ij} =$ the technical requirement of the jth decision variable for ith ressource or constraint

$G_k =$ the predetermined goal of the kth goal constraint

$B_i =$ the ith absolute constraint level

d_k^- and $d_k^+ =$ goal deviation variables which denote, respectively, under- and overachievement of the kth goal

d_i^- and $d_i^+ =$ absolute constraint deviation variables which denote, resspectively, under- and overutilization of the ith absolute constraint

Under such formulation, given any type of goal, the nonachievement of that goal is minimized by minimizing specific deviation variables. That is, we will desire to select x so as to:

(a) minimize d_k^+, if the goal constraint is denoted as

$$\sum_{j=1}^{n} g_{kj}x_j \leq G_k$$

(b) minimize d_k^-, if the goal constraint is denoted as

$$\sum_{j=1}^{n} g_{kj}x_j \geq G_k$$

(c) minimize $d_k^- + d_k^+$, if the goal constraint is denoted as

$$\sum_{j=1}^{n} g_{kj}x_j = G_k$$

Then the deviation variables at each priority level are included in the achievement function $r_q(\underline{d}^-, \underline{d}^+)$ and ordered in the achievement vector $\underline{a}$ according to their respective priority.

2. Absolute Constraints

In general, absolute constraints comprise production capacities, institutional, structural and functional absolute limitatations or restrictions as well as other assumptions. The absolute constraints comprising all items above can be expressed in the mathematical form below:

$$(2\text{-}2) \quad \sum_{j=1}^{n} b_{ij}x_j + d_i^- - d_i^+ = B_i; \quad i = 1, 2, \ldots, m.$$

[1] Notice that the notation $P_q r_q(\underline{d}^-, \underline{d}^+)$ should not be interpreted as either the product of P_q and $r_q(\underline{d}^-, \underline{d}^+)$ or a function of $r_q(\underline{d}^-, \underline{d}^+)$.

These constraints can be replaced, if necessary, by the appropriate formulations according to the constraint types.[2]

3. Goal Achievement Functions

A. *Expected Total Gross Margin*

The gross margin of an enterprise is its enterprise output less the variable costs attributable to it. To be regarded as variable costs in the gross margin sense, costs must satisfy two criteria: they must be specific to a single enterprise and vary approximately in proportion to the size of the enterprise. For crop production, the proportional specific variable costs consist primarily of seed, fertilizer, sprays, part of the fuel and repair costs, contract work and casual labor hired specially for that enterprise. For livestock production, concentrate feed, veterinary and medicine expenses, seed and fertilizer for grass, and some minor miscellaneous expenses are regarded as variable costs [2], [7].

Gross margins can probably be calculated without great difficulty. However, we are still faced with a problem: which gross margin data over time should be used in a static model for farm planning? While the model concerned is static, we live in a dynamic world: techniques and prices change constantly. In addition, some uncertain factors may cause variation in gross margin from year to year. Therefore, it is undesirable to use the data based on the previous one-year experiences, because they do not properly reflect the variation in gross margin due to the weather and market conditions [6]. And to use the average gross margin data based on an excessively long period is insufficient because they do not properly reflect the rapid development of techniques Hence we assume that it is adequate to use the average data based on recent five-year experiences.[3] During periods of serious inflation, however, they must

---

[2] Each equation of the absolute constraints can be replaced, according to the constraint type, by one of the following formulations:

$$\sum_{j=1}^{n} b_{ij}x_j \geq B_i, \text{ if } d_i^- \text{ should be zero}$$

$$\sum_{j=1}^{n} b_{ij}x_j \leq B_i, \text{ if } d_i^+ \text{ should be zero}$$

$$\sum_{j=1}^{n} b_{ij}x_j = B_i, \text{ if both } d_i^- \text{ and } d_i^+ \text{ should be zero}$$

Such transformation forms are of course unusual in LGP. But they can reduce the dimension of matrix used and consequently the required time and cost involved in solving the problem, if the problem is solved using an iterative algorithm.

[3] Taking a shorter period can generally reflect the trend (or level) of technical development better while selecting a longer period can reflect the variations in gross margins better. Therefore, it may be appropriate to take a five-year period because the length of period can reflect both of them to a certain extent.

be adjusted in constant prices.[4]

Now let us turn to setting the goal of total gross margin. Since the desire to maximize expected total gross margin is generally inexhaustible, the goal will be infinite. But the goal can be set by choosing an arbitrarily high level which is hardly attainable.

With the enterprise gross margins and the arbitrarily established goal, the goal achievement function for expected total gross margin can be formulated as follows:

$$(2\text{-}3) \quad \min \quad r_y(\underline{d}^-, \underline{d}^+) = d_y^-$$

$$\text{s.t.} \sum_{j=1}^{n} \bar{g}_{yj} x_j + d_y^- - d_y^+ = G_y$$

where $d_{y.}^-$ and d_y^+ denote, respectively, under- and overachievement of expected total gross margin goal, $\bar{g}_{yj}$ represents the sample mean gross margin per unit of the jth enterprise, and G_y is the goal of total gross margin. The goal achievement function is here represented by one variable, d_y^-.

B. Survival

An assumption on risk or uncertainty makes an important distinction between the two objectives of maximizing expected total gross margin and of ensuring survival: while the former assumes that the problem is free from risk or uncertainty, the latter assumes that the problem is closely linked with it. Without risk or uncertainty, the two objectives may no longer be in conflict.

Actually the problem surrounding risk or uncertainty is as complicated and difficult to deal with as that dealing with multiple objectives. Numerous techniques and decision models to embody this problem have been developed and applied to whole farm planning. Among them, most of the quadratic risk programming approaches are based on Markowitz's portfolio selection technique [27] [28]. Freund [11] made the first application of active stochastic programming to farm management. Wolfe [34] developed a somewhat different approach to make use of a parametric programming procedure. But, compared with linear programming, quadratic programming algorithms make heavy computational demands. Hence a number of attempts have been made to develop linear programming models that take account of the stochastic nature of enterprise gross

---

[4] The gross margin of the jth enterprise for farm planning in year t is calculated with the following equation:

$$g_{yj.t} = 1/5 \sum_{k=1}^{5} (g_{yj.t-k})/(p_{j.t-k}) = \bar{g}_{yj}$$

where $g_{yj.t-k}$ denotes the gross margin observed for the jth enterprise in year $(t - k)$ and $p_{j.t-k}$ represents the prices received by farmers in year $(t-k)$ for the products of the jth enterprise $(p_{j.t-1} = 1)$. And $\bar{g}_{yj}$ is the sample mean gross margin per unit of the jth enterprise.

margins in whole farm planning. McInerney [29] [30] and Hazell [13] among others attempted to incorporate game theory decision criteria into a programming formulation; Boussard and Petit [5] and Boussard [4] suggested an approach to use constraints on maximum admissible loss known as the focus-loss approach or maximum admissible loss approach; and Hazell developed a technique leading to a linear programming problem which incorporates the mean absolute deviation in place of variance as the measure of risk [9], [14], [15]. This is generally referred to as MOTAD-minimization of total absolute deviations.

However, all of these approaches seem to bear in mind a single objective function which combines both objectives of total gross margin or profit maximization and of risk minimization. To apply these approaches to the LGP model, it is desirable to modify them appropriately. Among the linear programming approaches mentioned above, the MOTAD approach seems to be more appropriate for a decision analysis view of the whole farm planning problem [1] [31]. It also readily permits the incorporation of assessed probabilities of occurrence of alternative states of nature.

Given an appropriate sample of enterprise gross margins from prior years, an unbiased estimate of the mean absolute deviation of expected total gross margin [17] is given by

$$(2\text{--}4) \qquad A = 1/w \sum_{h=1}^{w} |\sum_{j=1}^{n} (g_{yjh} - \bar{g}_{yj}) x_j|$$

where w shows the sample size, g_{yjh} denotes the gross margin observed for the jth enterprise in year h, and $\bar{g}_{yj}$ is the sample mean gross margin per unit of the ith enterprise. An equivalent, but computationally tidier approach posited by Hazell [14] is to work with the mean absolute value of negative deviations about the mean, estimated as

$$(2\text{--}5) \qquad D = A/2 = 1/w \sum_{h=1}^{w} y_h^-$$

where y_h^- shows the absolute value of the negative total gross margin deviation in year h around the expected return based on sample mean gross margins and is defined by

$$(2\text{--}6) \qquad y_h^- = |\min\{\sum_{j=1}^{n} (g_{yjh} - \bar{g}_{yj}) x_j, 0\}|$$

Following Hazell, the programming problem can be formulated as the minimization of the sum of the negative deviation variables, subject to the usual technical constraints and to a parametric constraint on expected total gross margin; i.e.,

$$(2\text{--}7) \qquad \min wD = \sum_{h=1}^{w} y_h^-$$

$$\text{s.t.} \sum_{j=1}^{n} \left(g_{yjh} - \bar{g}_{yj} \right) x_j + y_h^- \geq 0, \, h = 1, 2, \ldots ,w$$

$$\sum_{j=1}^{n} \bar{g}_{yj} x_j = \lambda, \, \lambda = 0 \text{ to unbound}$$

where λ denotes a parametric constant on expected total gross margin.[5]

This original MOTAD model can be alternatively formulated as the maximization of expected total gross margin with a parametric constraint on the sum of the negative deviations [1]. That is,

$$(2\text{-}8) \qquad \max \sum_{j=1}^{n} \bar{g}_{yj} x_j$$

$$\text{s.t.} \sum_{j=1}^{n} \left(g_{yjh} - \bar{g}_{yj} \right) x_j + y_h^- \geq 0, \, h = 1, \ldots, w$$

$$\sum_{h=1}^{w} y_h^- \leq \beta, \, \beta = 0 \text{ to unbounded}$$

where β represents a parametric constant on the sum of the negative deviations.[6]

Another slightly modified but equivalent form of MOTAD sets the parametric term into the objective function; i.e.,

$$(2\text{-}9) \qquad \max \sum_{j=1}^{n} \bar{g}_{yj} x_j - \gamma \sum_{h=1}^{w} y_h^-$$

$$\text{s.t.} \sum_{j=1}^{n} \left(g_{yjh} - \bar{g}_{yj} \right) x_j + y_h^- \geq 0, \, h = 1, \ldots w$$

where γ dentoes a risk aversion parameter.

In the last type of models, expected total gross margin with a certain level of probability is maximized. So this type of model seems to be more in agreement with our desire to incorporate the survival objective—ensuring the family's survival with a certain level of probability—into the LGP model under consideration for farm planning of semi-subsistence farms.

But the risk aversion parameter γ in the formulation (2–9) is not clear in representing the degree of risk aversion. Therefore, we must transform the parameter into another appropriate form in order for its concept to be clear. For this purpose, it is desirable to extend this model to the

---

[5] The objective function is also subject to the usual technical and non-negative constraints. In the formulation, however, the description of the constraints was omitted for simplicitiy's sake. This condition holds for models (2–8) and (2–9) as well.

[6] We may say that the LOTAD (Limitation On Total Absolute Deviation) model proposed by LOW [26] belongs to this type of MOTAD models. The only difference between the two models lies in that the LOTAD model establishes as the maximum admissible loss the following form instead of β in the parametric constraint of the formulation (2–8):

$$\sum_{j=1}^{n} \bar{g}_{yj} x_j - Z_0$$

where Z_0 represents a parametric constant.

case where the sample of gross margin observations is regarded as a set of states of nature, each of which is assigned a probability of occurrence, p_h, $h = 1$, . . . ,w with $\sum_{h=1}^{w}(p_h) = 1$.[7] Then,

$$(2\text{-}10) \qquad D = A/2 = \sum_{h=1}^{w} p_h y_h^-$$

And as shown by Hazell [14], justification for the use of A as a measure of risk is based on the fact that an unbiased estimate of the population variance is given by $A^2 \{\pi w/2(w-1)\}$ when the population is normally or approximately normally distributed [3], [16], [24]. Then the stochastic term in the objective function of the formulation (2–9) can be transformed into a weighted sample standard deviation of expected total gross margin as follows:

$$(2\text{-}11) \qquad \gamma \sum_{h=1}^{w} y_h^- = \phi s_y = \phi \Omega A = 2\phi \Omega D$$

$$= 2\phi \Omega \sum_{h=1}^{w} p_h y_h^- \ \ldots \ldots \ldots \text{.[8]}$$

$$= 2\phi \Omega w^{-1} \sum_{h=1}^{w} y_h^- \ \ldots \ldots \ldots \text{.[9]}$$

where s_y denotes the sample standard deviation of expected total gross margin,[10] ϕ is the parametric weight of risk, and Ω represents a constant which converts the sum of sample deviations to the sample standard deviation, defined as $\{\pi w/2 \ (w-1)\}^{1/2}$. In this transformation, the risk parameter in the formulation (2–9) was exchanged for a new parameter ϕ and a constant, $2\Omega w^{-1}$; consequently, the objective function of the formulation (2–9) can be replaced by the following equation [12]:

$$(2\text{-}12) \qquad \max \sum_{j=1}^{n} \bar{g}_{yj} x_j - 2\phi \Omega w^{-1} \sum_{h=1}^{w} y_h^-$$

Such transformations better clarify what the risk parameter means, for while a certain value of the parameter γ in the formulation (2–9) cannot offer us a concrete meaning, the new parameter ϕ in the formulation (2–12) can. That is, the latter corresponds with the admissible probability to risk or uncertainty when the population is normally or approximately normally distributed. In other words, under the assumption of normal distribution of population, the level of ϕ can be derived by making

---

[7] If a sample of enterprise gross margin from prior years is used, all of p_k will have the same value 1/w.

[8] From the foumulation (2–10).

[9] See footnote 7.

[10] The sample standard deviation of expected total gross margin, s_y, is different from that of the total gross margin observed in prior years. Rather, it varies with expected total gross margins considered in the process of solving the problem.

use of the tables for the t-distribution found in most statistics textbooks[11] if the admissible probability to risk or uncertainty is determined.[12]

The next step for formulating the survival achievement function is to set the goal level of survival. The level depends primarily on the average living standard of the society and the number of family members. The tradition, customs and habits of society also tend to affect the level. Food, clothing, fuel, light, and housing costs among others are generally regarded as inevitable living expenditures. In the society of semi-subsistence farmers, a contribution to celebrations or a donation to the ritual of mourning tends also to be considered inevitable. Education is usually regarded as a luxury consumer good, but the farmers tend to regard it as a necessary good. To them, their children's education may be as important as food, clothing, fuel, light and housing. Consequently, all items mentioned above are correctly included in the survival goal. Hence, the recent real living expenditure data about the above-mentioned items may be used to set the survival goal.[13]

With the established goal, G_s, and the formulations (2–9) and (2–12), the survival goal achievement function can be now formulated. That is,

$$(2\text{–}13) \qquad \min r_s(\underline{d}^-, \underline{d}^+) = d_s^-$$

$$\text{s.t.} \sum_{j=1}^{n} \bar{g}_{yj} x_j - 2\phi\Omega w^{-1} \sum_{h=1}^{w} y_h^- + d_s^- - d_s^+ = G_s$$

$$\sum_{j=1}^{n} (g_{yjh} - \bar{g}_{jh}) x_j + y_h^- \geq 0, h = 1, \ldots, w$$

where d_s^- and d_s^+ denote, respectively, under-and overachievement of the survival goal.

C. Financial Self-Support

Semi-subsistence farmers do not in general like to borrow money from others unless it is absolutely necessary. Hence, it is natural for financial self-support to be regarded as one of the important objectives in farm planning for semi-subsistence farms.

Types of credit available to the farms can be broadly classified into

[11] For a sample as small as 10, the sample standard deviation s_y will not be an accurate estimate of that of population σ_y; consequently, a serious error may be introduced in the value of z in replacing σ_y by its sample estimates s_y. See HOEL [18]. Hence, for a small sample, it is desirable to use Student's t-distribution rather than the standard normal distribution.

[12] The admissible probabilities to risk or uncertainty mean the subjective limits which individual farmers are willing to permit. Therefore, they should be determined by individual farmers. Suppose, they are 5%, 10%, 20% and so on. Then the corresponding values of the parameter ϕ will be, respectively, 2.132, 1.533, 1.190, .941 and so on, when the sample is as small as 5.

[13] In this context of survival, the survival costs already set in the absolute constraints should be deducted from total survival costs.

two categories: production credit from the public sector including agri-cutural cooperatives and general credit from the private sector including private banks. The public production credit is mainly made to encourage or stimulate production of specific goods, and is consequently closely linked with specific enterprises. Interest rates tend to be relatively low. On the other hand, private credit is not directly linked with any specific production activities, and interest rates are relatively high.

In order to incorporate these different types of credit into one achieve-ment function, we need to devise a commensurable unit. For our purpose of formulating the achievement function of financial self-support, interest rates are a proper commensurable unit of the different types of credit. Thus, the objective of financial self-support is replaced by the objective of no interest payments. The achievement function can be mathematically formulated as follows:

$$(2\text{--}14) \quad \min r_r(\underline{d}^-, \underline{d}^+) = d_r^+$$
$$\text{s.t.} \sum_{j=1}^{n} g_{rj}x_j + d_r^- - d_r^+ = G_r$$

where g_{rj} denotes the interest rate of credit j, and d_r^- and d_r^+ are respec-tively under-and overachievement variables for the total interest payment goal or the financial self-support goal. G_r indicates the total interest pay-ment goal whose value is generally zero.

D. Leisure

How much famrers desire to reduce their labor demand may differ from farm to farm, but we can assume that the desired labor supply is zero. The goal achievement function of leisure during the peak seasons can then be formulated as follows:

$$(2\text{--}15) \quad \min r_l(\underline{d}^-, \underline{d}^+) = d_l^+$$
$$\text{s.t.} \sum_{j=1}^{n} g_{lj}x_j + d_l^- - d_l^+ = G_l$$

where g_{lj} denotes the required labor of enterprise j during peak seasons and d_l^- and d_l^+ represent respectively under- and overachievement of the goal of total labor supply, G_l.

4. Achievement Function Vector

With the survey results and the achievement functions (2–3), (2–13), (2–14) and (2–15), the achievement function vector can be formulated as follows:

$$(2\text{--}16) \quad a = \{P_1r_1(d^-, d^+), P_2r_2(d^-, d^+), \ldots, P_4r_4(\underline{d}^-, \underline{d}^+) = (P_1d_s^-, P_2d_y^-, P_3d_r^+, P_4d_l^+)$$

However, the value trade-off assumption of goal programming may be too strict. It is very sensitive to the ordinal ranking and the goal vector set for the objectives given by the decision maker. That is, the model

assumes implicitly a value trade-off such that in minimizing $r_{q+1}(\underline{d}^-, \underline{d}^+)$, the value preference for r_q over r_{q+1} is zero if $r_q \le r_q^*$ and infinity if $r_q > r_q^*$, where r_q^* is the minimum value obtained for r_q when (q-1) previous achievement functions have already been minimized.

In fact, farmers may attempt to substitute a certain percentage of the achievable level of the higher ranked objective with greater achievement of the lower ranked objective. That is, farmers may be willing to increase expected total gross margin by sacrifying a certain percentage of the achievable level of the survival goal, to substitute a certain level of loss of the achievable expected total gross margin with some decrease in the involved indebtedness, and to replace some sacrifice of the achievable degree of financial self-support with curtailed work during the peak seasons.

This problem may be dealt with by making use of the method which Waltz proposed for reducing the sensitivity of lexicographic utility orderings [32]. The method will be later described in dealing with the algorithm of the LGP model.

5. The Farm Planning Model for Semni-Subsistence Farms
Now, we can formulate the comprehensive farm planning model for semi-subsistence farms with the goal achievement function vector (2–16), the absolute constraints (2–2), the goal constraints (2–3), (2–12), (2–14) and (2–15), and the non-negative and zero constraints as follows:

(2–17) Find $x = (x_1, x_2, \ldots, x_n)$ so as to minimize
$$\underline{a} = (P_1 d_s^-, P_2 d_y^-, P_3 d_r^+, P_4 d_l^+) \text{ subject to}$$

$$\sum_{j=1}^n \bar{g}_{yj} x_j - 2\phi\Omega w^{-1} \sum_{h=1}^w y_h^- + d_s^- - d_s^+ = G_s$$

$$\sum_{j=1}^n \bar{g}_{yj} x_j + d_y^- - d_y^+ = G_y$$

$$\sum_{j=1}^n g_{rj} x_j + d_r^- - d_r^+ = G_r$$

$$\sum_{j=1}^n g_{lj} x_j + d_l^- - d_l^+ = G_l$$

$$\sum_{j=1}^n (g_{yjh} - \bar{g}_{yj}) x_j + y_h^- \ge 0, \, h = 1, 2, \ldots, w$$

$$\sum_{j=1}^n b_{ij} x_j \lesseqgtr B_i, \, i = 1, 2, \ldots, m$$

$$\underline{x}, \underline{d}^-, \underline{d}^+ \ge 0$$

$$d_k^-, d_k^+ = 0$$

That is, an LGP model was developed for the farm planning of semi-subsistence farms. In the model, a goal achievement vector is minimized subject to the various constraints such as goal, absolute, stochastic, non-negative and zero constraints.

III. Solution Algorithm

The algorithms available for solving general linear goal programming models can be classified into two types. One is the iterative (or sequential) algorithm using standard simplex computer codes [10]. The other is the multiphase (or modified) simplex algorithm [20] [25] which is in essence simply a multiphase extension of the two-phase simplex algorithm [33]. In the former algorithm, the accomplishment level of the goal deviation variables as obtained for the higher priority level has to be included as a new rigid constant for each lower level priority model. But an outstanding advantage of the algorithm lies in its use of a commercial simplex package already available. On the other hand, the multiphase algorithm generally requires fewer pivots and eliminates the need for the construction of new constraints at each sequence [21].

In the application of this model, the iterative algorithm is desired. The algorithm, in addition to making use of the commercial simplex packages, has another important advantage in that the method proposed by Waltz can be easily used to reduce the sensitivity of the predetermined goal vector and the ordinal ranking given by the decision maker. According to Waltz's method, after the first objective is optimized, the second objective is opitmized subject to keeping the first objective within a certain percentage (α_1) of its optimum. The third objective is then optimized keeping the first two within a certain percentage (α_2) of the optimum values found in the previous steps, and so on.

The iterative algorithm combined with the method proposed by Waltz is portrayed in Figure 3–1. That is,

Step 1: Set q = 1 and proceed to step 2.
Step 2: Minimize the achievement function, $r_q(\underline{d}^-, \underline{d}^+)$, subject to the related goal constraint and the other constraints.
Step 3: Examine the feasibility of the constraint set. If it is feasible, proceed to step 4. If not, the matrix must be adjusted. Go to step 8.
Step 4: Check if there is any other lower ranked achievement function. If it exists, the solution is intermediate. Proceed to step 5. If not, the solution becomes global. Go to step 8.
Step 5: Calculate the achievable goal (G_k^*) with the solution and reduce the sensitivity of the predetermined goals and ordinal ranking by relaxing the achievable goal as much as α_q. Proceed to step 6.
Step 6: Replace the achievement function and the relevant goal constraint according to the explicit form of the achievement function with an equation including the relaxed goal (G_k^{**}) as follows:

$$\sum_{j=1}^{n} g_{kj}x_j - d_k^+ = G_k^{**}, \text{ if } r_q(\underline{d}^-, \underline{d}^+) = d_k^-$$

$$\sum_{j=1}^{n} g_{kj}x_j + d_k^- = G_k^{**}, \text{ if } r_q(\underline{d}^-, \underline{d}^+) = d_k^-$$

FIGURE 1 FLOW DIAGRAM FOR THE ITERATIVE LGP ALGORITHM COMBINED WITH WALTZ'S METHOD
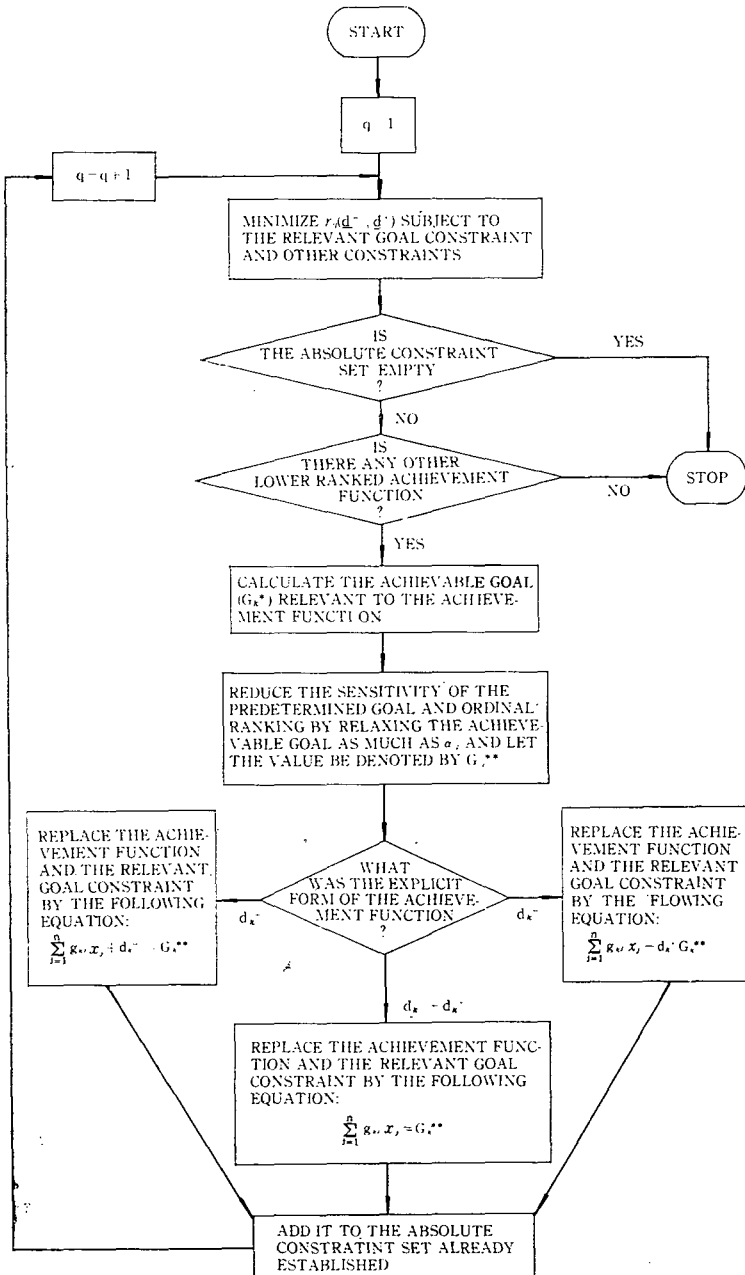
$$\sum_{j=1}^{n} g_{kj} x_j = G_k^{**}, \text{ if } r_q(\underline{d}^-, \underline{d}^+) = d_k^- + d_k^+$$

Add it to the absolute constraint set already established so as to become a new rigid constraint for the lower ranked achievement functions. Proceed to step 7.

Step 7: Set $q = q + 1$ and turn to step 2.

Step 8: Stop.

IV. Comparison of Farm Planning Results between LPG and Other Planning Methods

1. Data

In order to compare the farm planning results by the LGP model derived here with those by other methods, a village in a Korean hilly region was purposely selected as the sample. The sample village, Osan, located in the north-western part of Kyeongsangnam-do, Korea, consists of 60 households, of which 58 households do farming. These 58 farms were selected as sample farms, and a survey was conducted during the period from July to September 1980. And some secondary data were also added.

2. Some Assumptions

A. The Risk Aversion Parameter (ϕ)

The MOTAD model developed for formulating the LGP model includes a risk aversion parameter (ϕ) as well. The parameter reflects the decision maker's personal evaluation of potential consequences of risk (i.e., it corresponds with the admissible probability of risk). In other words, if the admissible probability to risk or uncertainty is determined by the farmer, the level of the risk aversion parameter can be derived by making use of the tables for the t-distribution found in most statistics textbooks under the assumption of a normal distribution of the population. The admissible probability is defined as the subjective limit to which the farmer is willing to submit. Therefore, the value will change with his personal evaluation of the potential consequences of risk.

Since the levels of subjective probability pursued by individual farmers are different according to their personal evaluations of potential consequences of risk, they should be determined by inquiring of individual farmers. In the application of the LGP model, however, it will be assumed that all farmers accept the same admissible probabilities of 10% to risk (i.e., the same subjective probabilities of 90%). Consequently, the risk aversion parameter will be assumed to be 1.533 for all sample farms.

B. The Trade-Off Parameters (α_q)

In the LGP model, multiple objectives are incorporated within an LP framework based on lexicographic utility orderings. However, the solution

is generally very sensitive to the goal vector and the ordinal ranking of the objectives given by the decision maker. To reduce this sensitivity, Waltz's method is incorporated in the model. The rationale of this method is that farmers may attempt to substitute greater achievement of the lower ranked objective for a certain percentage of the achievable level of the higher ranked objective.

The levels of trade-off parameters (α_q) are, however, determined by the individual farmers according to their personal evaluations of the consequences of each objective and the attainable levels of the goals. Therefore, they cannot be determined *a priori*. That is, whenever each iteration is terminated, the level of the related trade-off parameter should be determined by presenting the farmer the results of the iteration and inquiring his preferences. Nevertheless, this analysis was predicated on the assumption that the levels of preference are identically 5% in every iteration and for every farm.

3. Comparison of Expected Farm Planning Results

In order to show the differences between LGP and other planning methods, it is worthwhile to compare the expected farm planning results by the various methods. The methods to be compared with LGP are limited to a few simple methods: "by rule of thumb," the conventional single-objective LP model,[14] and a MOTAD model incorporating farmer's risk aversion behavior.[15] The comparisons are shown in Table 1.

The planning method, "by rule of thumb," denoted as alternative B in Table 1, implies what hitherto has been practiced by most Korean small farmers. Therefore, the numbers in the rows of the alternative B indicate historical data rather than farm planning results. Comparing LGP with the planning method, "by rule of thumb", the resulting goal achievement levels of financial self-support and labor supply are very similar. But the achievable levels of survival and expected total gross margin in LGP are shown to be higher than those for "by rule of thumb". This implies that reallocating resources currently used on the farms or undertaking a different combination of enterprises will allow a considerable increase in the achievable levels of survival and expected total gross margin.[16] A distinct difference between the two farm plans appears in the head of cattle to be raised. That is, the farm plan by LGP shows that, as long as the present technical possibilities and factor and product-cost relation-

---

[14] The LP model is incorporated in the goal achievement function of expected total gross margin in the LGP model.

[15] The MOTAD model compared here is that used in constructing the survival goal achievement function of the LPG model.

[16] This does not necessarily imply that the LPG model is irrelevant for farm planning for small farms. Rather, the differences will reflect deficiency in the farmers' measures. That is, they may overestimate the instability of special crop enterprise gross margins or the necessity for working cattle as draft power.

TABLE 1 COMPARISON OF FARM PLANS BETWEEN LPG AND OTHER PLANNING METHODS

Planning Methods	Achievement Level of Each Objective				Number of Cattle			
					Per Farm		Total	
	Survival	Total Gross Margin	Payment of Interest	Required Labor	Working Cattle	Beef Cattle	Working Cattle	Beef Cattle
	(000 won)	(000 won)	(000 won)	(hour)	(head)	(head)	(head)	(head)
A LPG[1]	1683.4	2013.2	42.8	1224.4	.03	.19	1.96	10.92
B By Rule of Thumb[2]	1541.1	1677.9	42.0	1223.5	.58	–	33.68	–
C Simple LP[3]	1545.7	2130.9	136.2	1326.7	.04	.58	2.52	33.74
D MOTAD[4]	1832.3	1979.4	137.5	1542.3	.76	–	43.94	–

Notes: 1) Linear goal programming model

2) Planning method "by rule of thumb" denotes what has been practiced on most Korean farms

3) Conventional LP model where expected total gross margin is maximized

4) MOTAD model where the stochastic nature of individual enterprise gross margins is incorporated, i.e., a farm planning model under risk or uncertainty.

ships remain constant, the number of working cattle currently raised on the farms will be reduced absolutely and some will be replaced by beef cattle.

Compared with LGP, the conventional single objective LP model maximizing expected total gross margin shows that the farms may possess substantial potential for increasing the expected total gross margin. It indicates, in particular, a considerable increase in beef production. However, the other objectives will not be fulfilled to the extent they are in the LGP model. Farmers may have to be ready to endure more anxiety about their survival, more credit, and more hard work during peak seasons. Considering the characteristics of Korean small farms, however, this is highly unlikely.

Alternative D is a MOTAD model for farm planning under uncertainty. According to this model, the goal for survival will be better achieved than that by the LGP model while expected total gross margin will be slightly reduced. And the farmers will have to borrow more money and work more diligently. The two models also show different results in the number and sorts of cattle to be raised on the farms. The MOTAD model shows that a greater number of cattle will be raised on the farms, but they will be working cattle exclusively. This model is generally regarded as too pessimistic.

The model that can best reflect the real world will be judged as the best one among them. However, from a theoretical point of view we can come to the conclusion that the LGP model is superior to the single-objective planning models insofar as the underlying assumptions are reasonable, for the single-objective models must be regarded as subsets of an LGP model.

V. Implications of the LGP Model

The LGP model developed here is still far from a panacea for the shortcomings of the traditional single-objective LP approaches. This model requires more information from the decision maker than that required in a single objective decision making model. The additional information must be accurate because the sensitivities are large. Despite these tedious requirements, the model seems to have several interesting and important implications.

First, the model can reflect the farmer's interests better in farm planning because it incorporates the farmer's multiple objectives explicitly in whole farm planning instead of implicit assumptions about these interests.

Second, the model can be solved despite its comprehensiveness by a relatively simple algorithm (i.e., by using a commercial simplex LP package already widely available).

Third, the model provides the farmer with important information

which can help him decide the trade-off parameters more easily and realistically.

Finanlly, the model and methods developed here can be applied to a wide field of actual and proposed decision making. That is, even though this model was developed particularly for farm planning of semi-subsistence farms, the model and the solving methods may be easily applied to other multi-objective decision making with slight modifications.

Bibliography

[1] Anderson, J. R., J. L. Dillon and J. B. Hardaker, 1977, *Agricultural Decision Analysis*, Iowa State University, Ames.

[2] Barnard, C. S. and J. S. Nix, 1979, *Farm Planning and Control*, Cambridge University, Cambridge.

[3] Barry, G. B., 1974, Portfolio Analysis under Uncertain Means, Variances and Covariances, *Journal of Finance*, Vol. 29, No. 2, pp. 515–22.

[4] Boussard, J. M., 1971, A Model of the Behavior of Farmers and Its Application to Agricultural Policies, *The European Economic Review*, Vol. 2, No. 4, pp. 436–61.

[5] Boussard, J. M. and M. Petit, 1967, Representation of Farmer's Behavior under Uncertainty with a Focus Loss Constraint, *Journal of Farm Economics*, Vol. 49, No. 4, pp. 869–80.

[6] Brandes, W., 1974, *Wie analysiere und plane ich meinen Betrieb?*, Paul Parey, Hamburg and Berlin.

[7] Brandes, W. and Woermann, 1971, *Landwirtschaftliche Betriebslehre (Band 2)*, Paul Parey, Hamburg and Berlin.

[8] Charnes, A. and W. W. Cooper, 1961, *Management Models and the Industrial Applications of Linear Programming*, Vol. 1 and 2, John Wiley, New York.

[9] Chen, J. T., 1971, A Linear Alternative to Quadratic and Semi-Variance for Farm Planning under Uncertainty: Comment, *American Journal of Agricultural Economics*, Vol. 53, No. 4, pp. 662–664.

[10] Dauer, J. P. and R. J. Krueger, 1977, An Iterative Approach to Goal Programming, *Operations Research Quarterly*, Vol. 23, No. 3, pp. 671–81.

[11] Freund, R. J., 1956, The Introduction of Risk into a Propramming Model, *Econometrica*, Vol. 24, No. 2, pp. 253–63.

[12] Hanf, C. H. and R. A. E. Müller, 1979, Risiko und Unsicherheit als angebotsbestimmender Faktor, *Agrarwirtschaft*, Jg. 28, H. 9, pp. 254–60.

[13] Hazell, P. B. R., 1970, Game Theory: An Extention of Its Application to Farm Planning under Uncertainty, *Journal of Agricultural Economics*, Vol. 21, No. 2, pp. 239–52.

[14] ———, 1971a, A Linear Alternative to Quadratic and Semi-Variance Programming under Uncertainty, *American Journal of Agricultural Economics*, Vol. 53, No. 1, pp. 53–62

[15] ———, 1971b, A Linear Alternative to Quadratic and Semi-Variance Programming for Farm Planning under Uncertainty: Reply, *American Journal of Agricultural Economics*, Vol. 53, No. 4, pp. 662–65.

[16] Hazell, P. B. R. and P. L. Scandizzo, 1974, Competitive Demand Structures

under Risk in Agricultural Linear Programming Models, *American Journal of Agricultural Economics*, Vol. 56, No. 2, pp. 235–44.

[17] Herry, E. M. J., 1965, Confidence Intervals Based on the Mean Absolute Deviation of a Normal Sample, *Journal of American Statistics Association*, Vol. 60, No. 309, pp. 257–70.

[19] Hwang, C. L. and A. S. Masud, 1979, *Multiple Objective Decision Making: Methods and Application*, Springer-Verlag, Berlin-Heidelberg-New York.

[20] Ignizio, J. P., 1976, *Goal Programming and Extentions*, D. C. Heath, Lexington.

[21] ———, 1978, A Review of Goal Programming: A Tool for Multiobjective Analysis, *Journal of The Operational Research Society*, Vol. 29, No. 11, pp. 1109–19.

[22] Ijiri, Y., 1956, *Management Goals and Accounting for Control*, North-Holland, Amsterdam.

[23] Kang, B. S., 1982, A Linear Goal Programming Model for Farm Planning of Semi-Subsistence Farms and Its Application to Small Farms and Their Beef Production in the Hilly Region of Korea (Unpublished), Ph. D. Thesis.

[24] Klein, R. W. and V. S. Bava, 1976, The Effect of Estimation Risk on Optimal Portfolio Choice, *Journal of Financial Economics*, Vol. 3, pp. 215–31

[25] Lee, S. M., 1972, *Goal Programming for Decision Analysis*, Auerbach, Philadelphia.

[26] Low, A. C. R., 1974, Decision Making under Uncertainty: A Linear Programming Model of Peasant Farmer Behavior, *Journal of Agricultural Economics*, Vol. 25, No. 3, pp. 311–22.

[27] Markowitz, H., 1952, Portfolio Selection, *Journal of Finance*, Vol. 7, pp. 77–91.

[28] ———, 1959, *Portfolio Selection: Efficient Diversification of Investments*, John Wiley, New York.

[29] McInerney, J. P., 1967, Maximin Programming: An Approach to Farm Planning under Uncertainty, *Journal of Agricultural Economics*, Vol. 18, No. 2, pp. 279–90.

[30] ———, 1969, Linear Programming and Game Theory Models-Some Extensions, *Journal of Agricultural Economics*, Vol. 20, No. 2, pp. 269–278.

[31] Thomson, K. J. and P. B. R. Hazell, 1972, Reliability of Using the Mean Absolute Deviation to Derive Efficient E, V Farm Plans, *American Journal of Agricultural Economics*, Vol. 54, No. 3, pp. 503–06.

[32] Waltz, F. M., 1967, An Engineering Approach: Hierarchical Optimization Criteria, *IEEE Transactions on Automatic Control*, Vol. AC-12, No. 2, pp. 179–80.

[33] Wilde, D. J. and C. S. Beightler, 1967, *Foundation of Optimization*, Prentice-Hall, Engelwood Cliffs.

[34] Wolfe, P., 1959, The Simplex Method for Quadratic Programming, *Econometrica*, Vol. 27, No. 2, pp. 382–98.

빈 면