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AN INVESTMENT PROGRAMMING MODEL FOR RURAL COMMUNITY WATER SYSTEM CAPACITIES WITH PRICE-SENSITIVE DEMAND*

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Introduction

Problem Stalement

Current methods for determining capacity for rural water facilties have too frequently relied on rules of thumb such as multiplying a current rate of per capita water consumption by a projected level of population over some specified period of time. Several economic problems are associated with this procedure. First it assumes the demand for water is perfectly price inelastic; however, economic theory and recent empirical studies indicate that the demand for water is price responsive. Second, the rule of thumb leaves no room for adjusting to different rates of population growth or different discount (interest) rates when determining the optimum timing and size of initial capacity and additions to capacity. Yet in determining how large to build initial or increments to capacity studies have emphasized two basic factors which are nearly always in conflict: (1) it pays to build large increments to capacity because there are usually cost savings (economies of scale) involved in capacity size; and (2) the commitment of resources to a capacity that will not be used for a period of time is costly since future costs are more heavily discounted than present costs. Third, water facility capacity is a resource flow concept rather than a resource stock concept. There are hourly, daily and seasonal fluctuations in the consumptive demand for water. A system designed for peak demand periods will have excess capacity during non-peak demand periods. Reducing demand during peak periods, perhaphs through pricing policies, reduces the need for greater system capacity.

Rural community water systems financed by federal loan programs through the Farmers Home Administration (FHA) have been unable to plan for sufficient capacity to meet increases in water demand due to popu-

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lation growth since the loan programs can consider only a fixed multiple of the existing population at the time of loan initiation. As a result many rural water systems financed by the loan programs must increase capacity after relatively short periods of operation, especially in fast growing areas.

Purpose and Objectives

The purpose of this research was to provide information for the planning and management of rural water systems in Oklahoma. The primary objective was to demonstrate an improved community services planning model by incorporating intertemporal and attitudinal correlates with decisions on investments in rural water services. Data derived from sample information on rural systems in Oklahoma were used as inputs in the planning model to determine optimum level in system capacity, level of operation and consumer satisfaction. Specific objectives were:

1. To review pertinent models of water resource investment planning.
2. To develop programming models related to optimum timing and size of rural water system investments and optimum pricing of water resources.
3. To evaluate past public investments in rural water services using the programming models.

Review of Water Investment Planning Models

An extensive amount of literature on water resource investment planning and water allocation has developed over the past two decades. Most of these studies apply mathematical programming techniques to solving the regional water resource planning problems. The major approaches pertinent to this study may be divided into two groups. The first group is the dynamic, multi-period capacity models. These models generally consider a given set of possible investment projects (e.g. reservoirs, water treatment plants) and compute the minimum cost of sizing and sequencing (timing) of these investment decisions to meet a particular set of demands that vary over time. However, these studies usually attempt to meet demands that are not price-sensitive. In this sense, demands are perceived as requirements in the model.

The second group is the models that simultaneously consider the allocation and capacity expansion decisions in planning water resource systems. These studies are based on the critical assumption that water demand is sensitive to changes in price. In addition to reviewing these two minor groups of studies, pertinent work that takes excess capacity (caused by economies of scale and positive social discount rates) into consideration while planning water system development is also discussed.

Capacity Expansion Models

Some of the early models of investment timing and sequencing were presented by Marglin (1963). The sequencing of simple independent projects with fixed scale to meet demand projections at minimum cost were first addressed by Butcher, Haimes and Hall (1969). Erlenkotter (1973a) proposed a direct ranking approach for appropriate sequencing decisions.

Extension of the simple dynamic programming sequencing framework to incorporate capacity independence among (hydroelectric) projects was first proposed and demonstrated by Erlenkotter (1973b). Becker and Yeh (1974a, 1974b) considered the problem of project independence in developing water supply for a river basin. Their approach associates a "firm" yield with each reservoir configuration considered in their dynamic programming sequencing, timing and sizing model. This "firm" yield is determined by routing the most critical period flows through each candidate configuration. The complication of indepedent project scale decisions were addressed further by a sequence determination framework. Another approach developed by Martin (1975) utilized a dynamic programming technique coupled with a network-with-gains algorithm to determine the optimal capacity expansion policy for a surface water supply system. All of these dynamic programming models minimize cost of meeting a prespecified, price-insensitive, dynamic (changing over time) demand.

Another attempt at the joint treatment of scale and sequencing was made by Jokoby and Loucks (1972) in a three stage procedure. They used a static linear programming model to obtain the initial project scale decision. These projects, with scale fixed, are sequenced with dynamic programming. The final solution is then evaluated in a simultaneous model. Although this conjunctive use of planning models and simulation models is a useful approach, it still does not guarantee a global solution.

More recently, Steiner (1977) has formulated a mixed integer programming model to determine the capacity expansion of a regional water resources system. Although marginal water costs have been explicitly computed and used as basis for pricing water in this framework, it still treated the water demand as price-insensitive.

Water Pricing and Capacity Expansion Models

Riordan (1971a) was first to use a more general economic efficiency criterion to obtain a solution to the pricing-investment problem. In this work a price-sensitive demand for the output of the projects under consideration is introduced and a marginal cost pricing criterion is defined as required for economic efficiency. Riordan (1971b) later applied this model to an investment-pricing problem in an urban water supply facilities system using hypothetical cost and demand curves.

Cysi and Loucke (1971) also used dynamic programming and price-sensitive demand to argue that increasing block rates were welfare maximizing in the long run for water treatment facility planning. Regev and Schwartz (1973) used discrete time control theory to formulate an interregional water investment and allocation model. Seasonal prices were explicitly considered. The results are general, but not operationally computable. Regev and Lee (1975) also developed a planning model for a river basin development using dynamic programming methods. Their model was used to find the optimal timing and scheduling of reservoir projects in a river basin when the demand is price-sensitive. Haimes and Hainis (1974) proposed an operational framework by incorporating an input-output demand model with a dynamic programming scheduling algorithm for a regional water supply system.

More recently a price-sensitive investment model was developed by Moore (1977) as an extension of the work by Becker and Yeh on the sequencing, timing and sizing of project investment work (1974b). Armstrong and Willis (1977) also formulated and demonstrated an investment and allocation model for water resources planning. They used the generalized Bender's decomposition approach to solve the resulting nonlinear mixed integer programming model. Adapting the sequencing algorithm of Erlenkotter and Rogers (1977b), two general frameworks for investment planning with price-sensitive dynamic demand have been proposed and illustrated by Erlenkotter and Trippi (1976) and Erlenkotter (1977a).

Optimum Excess Capacity Model

All of the above models were demonstrated to achieve appropriate planning schedules of overall water resources allocation with relatively little attention to deriving optimum excess capacity of water supply facilities such as the size of water mains or capacity of storage tanks to meet price-sensitive, growing intertemporal water demand.

Lynn (1973) was one of the first to address the problem of optimal facility scale. His work was preceded, however, by Chenery (1952) who developed a simple model for determining the optimal excess facility expansion. Chenery's model was redefined and extended by Manne (1961) whose work has received much attention from civil engineers. However, a basic problem with Manne's model is that the mathematical expression for the optimal design period is an implicit function and in order to calculate optimal excess capacities, trial and error or numerical techniques are necessary. To overcome this limitation, Lauria, Donald and Schlenger (1977) presented an approximating equation by which optimal excess capacity design periods can be calculated directly. Whereas Manne's work is limited to capacity expansions, Thomas (1970) extended Manne's model by including the optimal scale of a system for which the level of demand exceeds the capacity of supply facilities at the beginning of the planning horizon. Thomas' model also was approximated by Lauria, Donald and Schlenger (1977). Although optimal excess capacity design periods have been explicitly computed, again the weakness of these models is that they do not have a global solution due to assuming water demand implicitly as price-insensitive.

Distinctive Aspects of This Study

In comparison with earlier studies, the approach developed here for planning a rural water supply system differs in several aspects. First, the optimum excess capacity for initial and expansion systems are computed as an upper limit of the system. Economies of scale of water supply facilities are incorporated at a given discount rate to attain the optiaml excess capacity design. Second, price-sensitive demands are considered in the model. They are used not only to indicate the social benefits of water supply but also to yield the socially optimal prices, reflecting the cost of investments and operation and maintenance. Third, investment in an existing rural community water system in Oklahoma under conditions of uncertain growth is evaluated by comparing that system against the optimal prices and excess capacity design resulting from the model presented in this research.

An Investment Programming Model for Rural Water System with Price-Sensitive Demand

A mathematical programming model is developed for planning rural water system capacity when consumer's water demand is price dependent. The proposed procedure consists of selecting the optimum capacity, sequencing and timing of water system investments. The water rate decision is determined endogenously such that discounted net social benefits are maximized.

First, the assumptions of the model are presented. Second, the specific configuration of the model is described. Third, computational considerations and solution strategies are discussed. Finally, the basic LP model is presented.

Assumptions of the Model

The model presented here is based upon a fundamental assumption not ordinarily considered in water resources capacity decision models. The assumption is that water demand is sensitive to changes in price. Furthermore, it is assumed that aggregate demand for water varies over time and can be described by a continuous growth rate. It is assumed that the price elasticity of demand is constant throughout the planning period. The price-sensitive demand is then used in determining the consumer's willingness-to-pay and the total benefits of a rural water system.

In addition to the above major assumptions in the model, the following assumptions are adopted to simplify application in planning optimum water system investment:

1. Water demand in year y is a function of price in that year and no other period.
2. Capital investment costs occur as a lump sum at the time of initial construction and for any addition to capacity.
3. The operations and maintenance (O and M) costs occur as a lump sum in each year of operation.
4. The capital investment costs for initial construction and any additions are a linear function of capacity and assumed to reflect economies of scale, i.e., the cost per unit of capacity is either constant or decreasing with increasing capacity.
5. The O and M costs are a linear function of output.
6. The annual social discount rate, r, is assumed to be constant over time.
7. Inflation effects on benefits and costs are not considered.
8. The planning horizon is chosen as 40 years which is the FHA's loan repayment period for community water systems and is assumed equal to the anticipated lifetime of the initial water system investment.

Formulation of the Model

The objective of the programming model is to maximize the total discounted net benefits from investments in rural community water systems. The approach is to maximize the difference between the discounted sum of the benefits from water consumption and the sum of the discounted costs of the water system made up of investment and operation and maintenance.

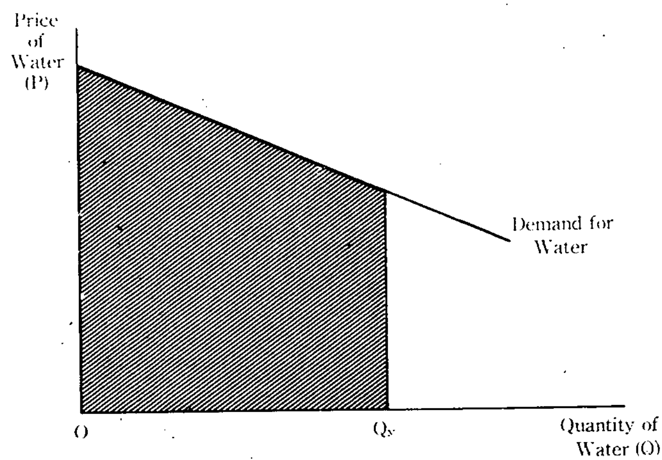
Benefit Function

The benefits associated with a given consumption of water in this analysis are measured by the consumers' willingness-to-pay which is denoted as the area under the demand curve up to a specific quantity demand level, say Q_y, in Figure 1. It is assumed that there is a one-to-one mapping of Q_y on $P_y(Q_y)$, the demand curve, and that when a value of Q_y is computed, the market-clearing price is also specified. For purposes of illustrating the approach, a linear demand is assumed in deriving the area under the curve although in the actual model a nonlinear demand curve is used.

Given the demand function for rural community water in year y the "willingness-to-pay" is denoted as:

$$f_y(Q_y) = \int_0^{Q_y} p_y(Q_y)\, dQ_y \tag{1}$$

FIGURE 1 Willingness-to-Pay for Q_y (Shaded Area) Quantity of Water



where Q_y is the community water demand in year y and P_y (Q_y) is the inverse demand function. For a given community the "willingness-to-pay" is discounted to the present and summed over the entire planning period using the annual social discount factor:

$$\alpha_y = \frac{1}{(1+r)^y}$$

where r is the social discount rate. This yields the following benefit function which appears in the objective function of the programming model:

$$TB = \sum_{y=1}^{Y} \alpha_y f_y (Q_y) \tag{2}$$

where Y is the length of the planning period in years.

Cost Function

Water system costs in the objective function consists of two major components. The first is the capital cost of the proposed water system. Since it is assumed that capacity reflects economies of scale, the capital cost function is concave. The capital cost function for the water system is denoted as S (S_τ), where S_τ is the capacity added in τ^{th} time unit (initial capacity is the addition from year zero).

Additions to water systems (excluding the initial capacity) have expected lifetimes that are assumed to be longer than the planning period. Capital costs are thus annualized over the expected lifetime of the addition and then discounted to the present for the period from the time of construction to the end of the planning period. The total present worth of these annualized capital costs are the costs that appear in the objective function. For the discount rate r, capital costs are converted to annual equivalent costs by applying the capital recovery factor β:

$$\beta = \frac{r(1+r)^m}{(1+r)^m - 1} \tag{3}$$

where r is the social discount rate and m is the expected lifetime of the capital investment.

For a given or proposed water system, the total discounted capital costs are:

$$TC = \sum_{\tau=1}^{T} \sum_{y=(\tau-1)\bar{y}+1}^{Y} \alpha_y \beta S(S_\tau) \tag{4}$$

where:

T = number of building time units in the planning period (if planning period is 40 and $\bar{y}$ is five years then T is 8)

$\bar{y}$ = number of years in a building time unit (additions to capacity are allowed once every $\bar{y}$ years, if necessary, in order to limit the number of decision variables and constraints in the model)

τ = index of building time unit, $\tau = 1, 2, \ldots, T$ (begin in year $y = 1, \bar{y}+1, 2\bar{y}+1, \ldots, (\tau-1)(\bar{y}+1)$.

The second cost component is for the expected system operation and maintenance (O and M). The O and M costs are defined as the annual costs for operation and maintenance of the system and are assumed to be a linear function of quantity of water delivered, (Q_y). It can be stated as cQ_y where c is the unit O and M costs and Q_y is the quantity of water delivered in year y.

The above O and M costs are discounted to the present and summed over the planning period. The final form of total discounted annual O and M costs is:

$$TO = \sum_{y=1}^{Y} \alpha_y cQ_y \tag{5}$$

Total Net Benefit

With equations (2), (4) and (5), the complete objective function for the programming model is expressed as follows:

$$Max.\ (TB - TC - TO) \tag{6}$$

which is to maximize equation (2) less equations (4) and (5).

Model Constraints

Having described the benefits and costs in the objective function, the necessary constraints required for a solution to the model are now expressed. The first set of constraints states that the quantity of water delivered in a specific time period cannot exceed total capacity built up to that period. This capacity constraint is stated as follows:

$$Q_y - \sum_{\tau=1}^{G} S_\tau \leq O \tag{7}$$

where $G = \lceil y/\bar{y} \rceil$, the ceiling of $y/\bar{y}$ which indicates the number of building time units up to year y.

The second set of constraints is the allocation constraint which requires that the actual water allocated in year y equals the water supplied in year y. This can be expressed as:

$$X_y - Q_y = O \tag{8}$$

where X_y is the quantity of water demanded in year y.

To assure that the capacity decision variable, S, can be established at most once during any building time unit, the following constraints are needed:

$$S_\tau - \bar{s}z_\tau \leq O \tag{9}$$

and

$$Z_\tau \leq 1 \tag{10}$$

where $\bar{s}$, a given value, is the maximum possible capacity (physical upper bound) of the water system and Z_τ is a zero-one decision variable representing the decision to add capacity in period $\tau(Z_\tau = 1)$ or not to add capacity in $\tau(Z_\tau = 0)$.

Finally, for solutions of this model to be meaningful, all above decisions are required to be non-negative.

Computational Considerations

The optimization model formulated above has a nonlinear objective function with several linear constraints. Since the main focus is to develop a solvable mathematical model, approximations are made to render the optimization model compatible with currently available computer techniques. Piecewise or grid linearization and fixed-charge approximation techniques are used to approximate the nonlinear objective function. The concave benefit function is linearized in the following manner. Suppose a linear demand curve is written as follows:

$$P(Q) = a + bQ \tag{11}$$

where price, P, is a function of quantity, Q. Then the area under the demand curve, B, can be expressed as follows:

$$B = \int_0^Q P(Q)\, dQ = Q(a + 0.5bQ) \tag{12}$$

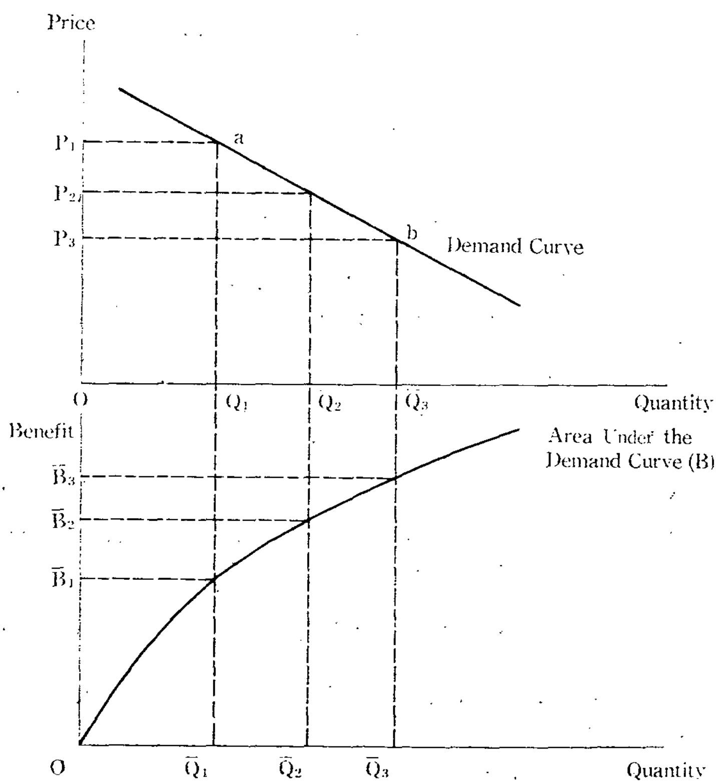
Now the objective function equation (6) can be rewritten as follows using equation (12):

$$\text{Max}: (Q(a + 0.5bQ) - S(S_\tau) - cQ) = NB \tag{13}$$

where *NB* is net social benefit. However, notice that equation (13) still contains a nonlinearity. Following Duloy and Norton (1975), this nonlinearity is removed through the use of the grid linearization technique. Grid linearization requires prior specification of a relevant range of values of the demand curve and the use of variable interpolation weights on the grid point. The interpolation weights become variables in the model and their values are jointly constrained by a set of convex combination constraints.

Implementation of the grid linearization technique is illustrated in Figure 2. Suppose that initially the demand curve defined in the price-quantity space passes throufih the point $(P_2, \bar{Q}_2)$. The relevant range of the demand curve is defined and truncated at points a and *b*. Then the relevant range of the demand curve is partitioned into segments $s = 1, \ldots,$ v. For each segment end point the parameters $\bar{Q}_s$ and $\bar{B}_s$ are defined to represent the cumulative known area under the aggregate demand curve for water.

FIGURE 2 GRID LINEARIZATION OF DEMAND AND BENEFIT FUNCTIONS



The quanitity of water used and the total area under the demand curve can be expressed as a weighted combination of $\tilde{Q}_s$ and $\bar{B}_s$ respectively.

$$Q = \sum_{s=1}^{V} \tilde{Q}_s W_s \tag{14}$$

$$B = \sum_{s=1}^{V} \bar{B}_s W_s \tag{15}$$

where W_s is a weight variable. The non-negative interpolation weight variables are defined such that $\sum_{s=1}^{V} W_s \leq 1$. Notice here that no more than two consecutive points on the quantity axis will enter the optimal basis.

For the capital investment cost function, a fixed charge (set-up cost) approximation approach is used. For example, the capital investment cost $S(S_\tau)$ becomes (see Figure 3):

$$S(S_\tau) = fZ_\tau + KS_\tau \tag{16}$$

where

f = fixed charge of the capital cost function, $S(S_\tau)$
K = slope of the capital cost function
Z_τ = binary decision variable

The Basic LP Model

To reduce the dimensions of the LP model, a five year decision time unit, τ, is used instead of an annual time unit, y. Thus, new discount rates, d_τ, and growth rates, h_τ, are computed which cover five year periods. Also, utilizing the grid linearization described, the basic linear programming model can be stated as follows:

FIGURE 3 Fixed Charge Capital Cost Function

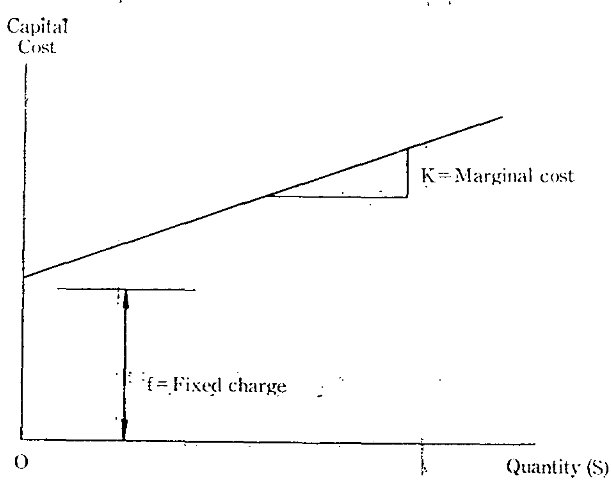


TABLE 1 INITIAL LP TABLEAU (2 PERIODS ONLY)

		RHS	O_1	S_1	Z_1	X_{11}	..	X_{1v}	Q_2	S_2	Z_2	X_{21}	..	X_{2v}
Max	E.Q	a	$-d_1c$	$-\sum_{y=1}^{Y}\alpha^{y-1}\beta K$	$-\sum_{y=1}^{Y}\alpha^{y-1} f$	d_1B_{11}	..	d_1B_{12}	$-d_2c$	$-\sum_{y=\bar{y}+1}^{Y}\alpha^{y-1}\beta K$	$-\sum_{y=\bar{y}+1}^{Y}\alpha^{y-1}\beta f$	d_2B_{21}	..	d_2B_{22}
WBAL	L.E.	o	—1			x_{11}	..	x_{1v}						
CAP	L.E.	o	1	—1										
CONV	L.E.	h_1				1	..	1						
INTEGER	L.E.	o		1	$-\bar{s}$									
WBAL	L.E.	o							—1			x_{21}	—	x_{2v}
CAP	L.E.	o		1					1	—1				
CONV	L.E.	h_2										1	..	1
INTEGER	L.E.	o								1	$-\bar{s}$			

$$MAX\ NB = \sum_{\tau,s} d_\tau (\bar{B}_{\tau s} W_{\tau s} - cQ_\tau) - \beta \sum_{\tau} \sum_{y=(\tau-1)\bar{y}+1}^{Y} \alpha_y (KS_\tau + fZ_\tau) \tag{17}$$

subject to
water balance equation (WBAL)

$$-Q_\tau + \sum_s Q_{\tau s} W_{\tau s} \leq O \tag{18}$$

system capacity constraint (CAP)

$$Q_\tau - \sum_{\tau=1}^{G} S_\tau \leq O \tag{19}$$

convex combination constraint (CONV)

$$\sum_{s\tau} W_{\tau s} \leq h \tag{20}$$

integer constraint (INTEGER)

$$S_\tau - sZ_\tau \leq O \tag{21}$$

A portion of the initial LP tableau (covering two periods) is presented in Table 1.

Results of the Programming Model

Results of the mixed integer programming model with water demand and supply data derived by Myoung (1982) are presented and discussed. The effects on community water system investments from varying parameters such as the growth rate and discount rate are investigated.

Since some of the coefficient (for example, price elasticity of demand, discount rate and growth rate) used in the planning model are subject to variability, a comprehensive sensitivity analysis of the most likely combinations of input parameters was performed. Such analyses should provide more insights into the usefulness of the proposed model for decision making purposes. A number of computer runs were made to explore the impact of each parameter on the benefit-maximizing investment function and the resulting water rates. The purpose is to show how sensitive water rates and investment decisions are to the discount rate and growth rate for a community's water system.

Base Results

The base results consist of an optimal capacity expansion schedule for a water system growing in the number of customers at eight percent annually, the operating level of the water system over time in association with the optimal investment schedule, and the water rates at which the consumer's demands are satisfied for varying discount rates. The operating levels actually imply a set of facility policies. The optimal solutions of the base results are for the average size community of the sample survey administered by Myoung (1982).

Optimal Capacity Investment Schedule

The average annual growth rate of the sample of rural water districts was eight percent per year. The optimal investment decisions for the average size community at initiation of water system services with expected eight percent per year growth are shown in Table 2. The solutions indicate that the initial system should be built at capacities of 136.9 mgy,[1] 108.7 mgy, and 93.8 mgy if one percent, three percent, and five percent discount rates are applied, respectively. According to the schedule of solutions these initial capacities are maintained through time unit three (15 actual yearrs in the model) and then new facilities are added at the beginning of time unit four. The size of added capacities beginning with time unit four are 179.5 mgy, 187.2 mgy, and 162.5 mgy, respectively, for the associated discount rates. The solutions also indicate that beginning with time unit six and until the end of the planning period new additions are made for every time unit. This is because the eight percent growth in the later time units bring more capacity requirements than the early time units. In other words, capacity should be added every five years to meet eight percent annual growth for the given discount rates. Total capacities built during the entire planning period are 1320.5 mgy, 1194.7 mgy and 1003.5 mgy, respectively.

Optimal solutions associated with the higher discount rates show that water systems are not build in time unit one even though there is a demand for water. In other words, the construction of water systems should be delayed until time unit two if the discount rate is seven percent and time unit four if the discount rate is nine percent. If the discount rate goes up to 15 percent, no water system is optimum under the model conditions. That is, the expected present worth of the cost (building and operation) of the system is greater than the expected present worth of the benefit it will provide regardless of when it is built (given the discount rate is 15 percent).

The programming results correspond with the theory discussed earlier that one of the factors determining optimal capacity is the social discount rate. Suppose the discount rate is zero. Then, it would be perfectly sensible to spend a dollar now in order to save a dollar's worth of costs either in the next time period or ten years from now, or 100 years, thus, the limit to the size of capacity is dependent only on the scale factor. With a positive discount rate, however, to save a dollar's worth of cost in a fu-

[1] mgy is million gallons per year.

TABLE 2 Optimal Capacity[a] Investment Schedule From the Basic Results at Eight Percent Growth

Discount Rate (percent)	Objective Value ($)	Building Time Unit								
		1	2	3	4	5	6	7	8	Total
1	5,534,429	136.9	—	—	179.5	—	295.2	287.6	421.3	1320.5
3	2,519,708	108.7	—	—	187.2	—	257.4	260.2	381.2	1194.7
5	1,062,444	93.8	—	—	162.5	—	208.5	218.5	320.2	1003.5
7	372,982	—	118.2	—	—	226.8	—	249.5	278.9	873.4
9	85,317	—	—	—	215.3	—	—	292.0	237.7	745.0
15	—	—	—	—	—	—	—	—	—	—

[a] Amount of system capacities in mgy.

ture time period we only need to spend less than a dollar now. Therefore, under given economies of scale if the discount rate is low the size of optimal capacity is relatively large whereas if the discount rate is high the size of optimal capacity is relatively small.

Optimal Water Supply Scdedule

There are two major factors which directly influence the short run level of water supply: size of capacity and growth in water demand. It is reasonable to say that an increse in number of customers will result in an increase in water supplied as long as excess capacity exists. However, how fast water supply should be increased depends mainly on the price elasticity of demand for water and the system's growth rate. Once water supply reaches the maximum capacity, to increase supply requires the next addition to capacity.

The optimal water supply schedule for the average size community in the sample with an eight percent growth rate during the planning period is presented in Table 3. As in the case of optimal investment, the various discount rates show the sensitivity on optimal water supply. For the case of a one percent discount rate the optimal water supply increases significantly from time unit one to time unit eight. Optimal water supply increases from one time unit to the next time unit except for time unit three which is the same as that of time unit two. This is because the system reaches its maximum capacity in time unit two and additional capacity is not optimum until time unit four. It is noted that the increase of water supply in the later time units are relatively greater than those of the earlier time units. This is explained by the compounding effect of an eight percent growth rate during the whole planning period. That is, eight percent growth in earlier time units results in relatively smaller net increases in number of customers than is the case for later time units. In fact, it is probably not realistic to assume that the water system grows at a constant rate during the whole planning period, i.e., eight percent. A more realistic assumption would be for water systems with fast growth at the beginning

TABLE 3 OPTIMAL WATER SUPPLY[a] SCHEDULE FROM THE BASIC RESULTS AT EIGHT PERCENT GROWTH

Discount Rate	Water Supply Level for Each Time Unit							
(percent)	1	2	3	4	5	6	7	8
1	93.8	136.9	136.9	297.3	316.5	611.6	899.2	1320.6
3	93.8	108.7	108.7	295.9	295.9	553.3	813.5	1194.6
5	93.8	93.8	93.8	256.3	256.3	464.8	683.3	1003.5
7	—	118.2	118.2	118.2	345.1	345.1	594.6	873.1
9	—	—	—	215.3	215.3	215.3	507.3	754.0
15	—	—	—	—	—	—	—	—

[a] Amount of water supplied in mgy.

and then slower growth during the remaining part of the planning period. Of course, the specific rate of growth depends upon the environment of individual systems.

The water supply schedule also includes solutions for various discount rates. As observed in the optimal capacity schedule, a system's water supply declines as the discount rate increases. Again there is no water supply in time unit one for the seven percent discount rate, time unit one, two and three for the nine percent discount rate, and the whole planning period for the 15 percent discount rate because no water capacity was built for these time units.

Optimal Water Rate Schedule

Optimal solutions for capacity and water supply representing different growth and social discount rates are read directly from the output of the programming model. However, the model does not provide the optimal water rate schedule directly. The optimal water rate is computed by substituting water supply for each time unit into that unit's demand equation representing a particular growth situation. To do this, it is necessary to derive the demand equation for each time unit.

Using the estimated price elasticity of demand for water and the initial average price and quantity of water demanded for the sample of rural water districts, the general demand function in rural Oklahoma was derived (Myoung). The demand equation at zero time in Table 4 shows that if the water rate increases one dollar per mgy the quantity of water demanded will decrease about 15,000 gallons per year. The assumption is made that consumer response to price change is relatively constant during the planning period even though the water system measured in terms of number of users grows in future time units.

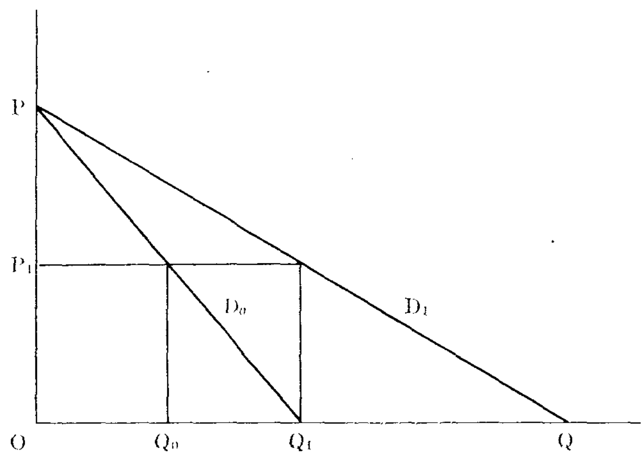
Growth of the water system on the price-quantity plane can be expressed by rotation of the initial demand curve as shown in Figure 4. Let D_0 represent the demand curve before growth (i.e. at time unit zero), whereas D_1 represents demand after growth at time unit one. The price-quantity relationship shows that if the price level is P_1, Q_0 amount of water is purchased by the given number of customers in a community (say 100 customers) at time unit zero. Assume that the number of customers increases to 200 at the end of time unit one—a 100 percent growth compared to the original number of customers. The amount of water purchased by 200 customers at time unit one would be Q_1 if the price level stays at P_1. Thus, by the assumption of constant consumer response, Q_1 should be exactly twice that of Q_0. Since this price-quantity relationship is true for each and every level of prices, the demand function for time unit one can be derived by using the information from the initial price-quantity relationship and growth in number of customers. Practically, this is derived for time unit one by dividing the slope of D_0 by its growth index.

TABLE 4 ROTATED DEMAND EQUATIONS FOR EACH TIME UNIT AT EIGHT PERCENT ANNUAL GROWTH RATE

Time Unit	Growth Index (h)	Demand Equations (Inversed)
0	1.00	p = 5300 – 68.6Q
1	1.47	p = 5300 – 46.8Q
2	2.16	p = 5300 – 31.9Q
3	3.17	P = 5300 – 21.7Q
4	4.66	P = 5300 – 14.8Q
5	6.85	P = 5300 – 10.0Q
6	10.06	P = 5300 – 6.8Q
7	14.79	P = 5300 – 4.7Q
8	21.72	P = 5300 – 3.2Q

P = price per mgy dollars.
Q = quantity of water demanded in mgy.

FIGURE 4 ROTATION OF DEMAND CURVE BY GROWTH



The demand equations for the different time units in Table 4 are derived in this manner—dividing the slope of the initial demand curve, 68.6, by the growth index in column two. For the Base Results, since a constant growth rate of eight percent per year is applied throughout the planning period, the demand curves become flatter and flatter as the system grows.

The optimal water rate schedule is computed by substituting the water supply into each time unit's demand equation. To analyze the optimal rate schedule, not only the relationship between optimal water supply and growth rate should be considered but also the optimal capacity schedule. This is because the water supply schedule is influenced by the optimal investment schedule. For example, in Table 5 the rate schedule for the one percent discount rate fluctuates from one time unit to another time unit depending upon timing of additional capacity. If there is pressure on capacity due to system growth it will result in addition of new capacity which allows an increase in water supply. The increased water supply brings the water rate down but not necessarily as low as if the system stayed on the same demand curve. The reason is that the slope of the new demand curve from which the optimal water rate is computed is now flatter than the previous demand curve.

In Table 2 for a one percent discount rate the initial capacity is 136.9 mgy but the actual water supply is 93.8 mgy at time unit one in Table 3. That is, 43.1 mgy excess capacity is reserved for future growth. Substituting 93.8 mgy amount of water supplied in the first time unit demand curve results in a water price of $910.20 per million gallons. In the second time unit, all of the existing capacity is utilized due to the system's growth. Therefore, again substituting the optimal water suply, 136.9 mgy into the second time unit's demand equation results in $932.90 per million gallons as the water rate which is only slightly higher than that of the first time unit. In the third time unit, there is another eight percent growth in the system but additional capacity has not come into the solution yet. Therefore, the amount of water supplied is restricted to the maximum capacity by raising the water rate. That is why the water supplied during the third time unit is the same as that of the second time unit but the water rate is significantly higher. Water rate is used as a means to allocate a given amount of water to more customers. In the fourth time unit there is another eight percent growth per year. Now the water system no longer relies strictly on the role of price to maintain existing capacity. Therefore a new capacity additon comes into the solution (see Table 2). With new additional capacity water supply increases and consequently the optimal water rate decreases. These interrelationships among growth rate, optimal capacity schedule, optimal water supply schedule, and optimal water rate continue until the end of the planning period for each discount rate.

Analysis for Alternative Growth Rates

Rural community water systems have shown substantial variability in growth (Myoung, 1982). The focus of this study was to maximize net soical benefits assuming decision makers knew the sytem's growth at the time of initial planning. This is seldom the case. Myoung (1982) presents an extensive analysis of program solutions for various growth rates, some varying within the planning period itself, and for different size water systems. Some general conclusions of that analysis are given here.

Consumers of rural water services in Oklahoma are price sensitive. Thus the price of water will affect the demand for water. For economic water rates should be set equal to the marginal cost of providing additional water. Thus, the objective of determining the price of water which maximizes social benefits must take into consideration the demand for water and the cost of supplying water.

TABLE 5 OPTIMAL WATER RATE[a] SCHEDULE FROM BASE RESULTS AT EIGHT PERCENT GROWTH

Discount Rate (percent)	Optimal Water Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	910.2	932.9	2329.3	900.0	2135.0	1141.1	1073.8	1074.1
3	910.2	1832.5	2941.2	920.7	2341.0	1537.6	1476.6	1477.3
5	910.2	2307.8	3264.5	1506.8	2737.0	2139.4	2088.5	2088.8
7	—	1529.4	2735.1	3550.6	1489.0	2953.3	2505.4	2506.1
9	—	—	—	2113.6	3147.0	3836.0	2915.7	2916.0
15	—	—	—	—	—	—	—	—

[a] Dollar per million gallons.

Water supply costs show significant economies of scale in rural water system investment and operation and maintenance. The growth analysis strongly supports the excess capacity model as a framework for planning optimum water system capacity. Failure to optimize on excess capacity may lead to under- or over-investment in community water systems and thus reduce social benefits due to inefficient allocation of resources. Under-investment for any particular community may force duplication of facilities (parallel lines) which could have been avoided if optimal capacity were planned from the beginning. Therefore, the objective of determining the optimum capacity of rural water systems which maximizes social benefits must incorporate expected growth in water demand as well as the economics of water supply.

Results of the mathematical programming model suggest the following policy decision criteria for planning rural water systems:

1. Price-sensitive consumer behavior should be considered in decisions of rural water services capacity design and water pricing.
2. The existence of economies of scale in water supply are important in determining optimum timing and size of water facility investment.
3. Predictions of growth are highly important in planning optimal water system capacity.
4. All of the above criteria should be considered simultaneously along with the discount rate in making global optimal water supply decisions for specific water districts.

Comparison of Net Social Benefits Between Actual and Optimum: The Case of Murray #1

To demonstrate the advantages of the optimal investment programming model for planning rural water systems, a comparison of program results was made with an actual system, Murray #1. Using the general demand equation for water and the actual water investment and supply records of

Murray #1, net social benefits were computed. Then net social benefits were computed using the optimal investment programming model and the actual rate of growth of Murray #1. Finally, the two net social benefits were compared.

Murray #1 water system started supplying water in 1967. The annual water demand, number of customers and investment record of Murray #1 are presented in Table 5. The amount of water demanded and the number of customers show dramatic increase since the system started operation. The initial number of users, 229 in 1967, increased to 934 in 1980 and results in a 12 percent annual growth rate. In addition to the initial investment, there were two expansion of capacities to meet growth of the system, 1973 and 1978.

It was assumed that the customers in Murray #1 have the same consumption behavior as explained by the general water demand equation. To reflect system growth, the general demand equation was rotated as explained previously. Specifically, the slope of the original demand equation was divided by the index of growth.

Using the rotated demand curves and the actual water demand, consumer benefits were computed. The revised demand equation and the gross benefits for each year are presented in Table 7. The gross benefits for each year are presented in Table 7. The gross benefits are discounted at five percent to compute the present worth of water consumption benefits. Also, Table 7 includes the present worth of actual O and M costs to run the water system each year and the present worth of gross benefits less the total present worth of O and M and capatal costs. The net social benefits equalled $204,478 for the actual Murray #1.

TABLE 6 Annual Water Demand, Number of Customers and Investment Record for Murray #1 Water System

Year	Water Demand (mgy)	No. of Customers	Index of Growth	Investment Record ($)
1967	18.2	229	100	314,745
1968	16.8	230	100	—
1969	17.8	243	106	—
1970	17.4	252	110	—
1971	17.3	268	117	—
1972	17.4	389	170	—
1973	24.0	475	207	66,000
1974	36.0	525	229	—
1975	40.7	566	247	—
1976	39.2	599	262	—
1977	38.8	654	286	—
1978	57.1	762	333	225,000
1979	63.4	859	375	—
1980	86.9	934	408	—

TABLE 7 Actual Benefits and Costs in Supplying Water for Murray #1 Water System

Year	Revised Demand Equations	Water Supply (mgy)	Gross Benefits ($)	Discounted Gross Benefits at 5% ($)	Discounted O&M Costs at 5% ($)	Discounted Capital Investment at 5% ($)
1967	P = 4840.2–189.4 Q	18.2	25,355	24,148	6,311	198,799
1968	P = 4840.2–189.4 Q	16.8	27,859	25,269	5,390	—
1969	P = 4840.2–178.7 Q	17.8	29,536	25,514	5,439	—
1970	P = 4840.2–172.2 Q	17.4	32,084	26,396	5,063	—
1971	P = 4840.2–161.9 Q	17.3	35,280	27,704	4,794	—
1972	P = 4840.2–111.4 Q	17.4	50,492	37,678	4,592	—
1972	P = 4840.2–91.5 Q	24.0	63,461	45,101	6,033	15,741
1974	P = 4840.2–82.7 Q	36.0	67,068	45,394	8,618	—
1975	P = 4840.2–76.7 Q	40.7	69,943	45,086	9,280	—
1976	P = 4840.2–72.3 Q	39.2	78,637	48,276	8,512	—
1977	P = 4840.2–66.2 Q	38.8	88,140	51,534	8,024	—
1978	P = 4840.2–56.9 Q	57.1	90,858	50,593	11,246	13,512
1979	P = 4840.2–50.5 Q	63.4	103,881	55,090	11,892	—
1980	P = 4840.2–46.4 Q	86.9	70,219	35,465	15,524	—
TOTAL				543,248	110,718	228,052

TABLE 8 OPTIMAL INVESTMENT, OPERATION LEVEL AND NET SOCIAL BENEFIT FROM THE PROGRAMMING LEVEL

Building Time Unit	Capacity (mgy)	Operation Level (mgy)	Net Social Benefit ($)
1	72.8	41.2	—
2	—	72.8	—
3[a]	55.2	128.0	—
Total	128.0		310,176[b]

[a] Adjusted to reflect four year time unit.
[b] Program does not permit allocation of net social benefits by time unit.

The optimum solution derived by the investment planning model is presented in Table 8. For the model solutions, the actual 12 percent growth rate is combined with the general demand equation and general O and M and capital cost functions. The optimum solution shows that 72.8 mgy capacity should have been built in the initial time unit and 55.2 mgy should have been added in the third time unit. The optimal supply schedule shows a significantly larger volume of water being supplied than for the actual system. The objective value generated by the optimal solution is $310,176 which is about 52 percent higher than that for the actual water system.

Several conclusions can be drawn from the results of these comparisons.

1. Decision makers underestimated growth of the water system and built too small an initial facility.
2. Because of an incorrect investment decision, the Murray #1 community lost considerable benefits which could have been gained if optimal decisions had been made.
3. Uncertainty relative to system growth may have been a major factor contributing to under-investments by the Murray #1 decision makers. The optimal programming model is a way to improve economic efficiency in decision making of water system investment but does not reduce the problem of uncertainty relative to system growth.

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