A REVIEW OF HOUSEHOLD COMPOSITION EFFECTS IN THE DEMAND FUNCTIONS*

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I. Introduction

The fact that the household consumption pattern is affected by several factors which may be economic or non-economic in nature is well known. The effects of household composition on family consumption has been discussed as the subject of extensive research (Syndenstricker and King, 1921; Nicholson, 1949; Prays and Houthakker, 1955; Forsyth, 1960; Cramer, 1973; Singh and Nagar, 1973; Muellbauer, 1974, 1975; Kakwani, 1977; Buse and Salathe, 1978). These papers describe ways in which allowances for the effects of household composition have been made in the formulation of Engel curves, based on the concept of consumer unit scale. This information aids in the specification and estimation of Engel functions, demand functions, and/or demand systems. However, a number of problems remain in the estimation and the interpretation of such models. This paper focuses on two issues that have not been well sufficiently examined in the literature. The first one concerns the interpretation of the consumer unit scales in the context of utility maximization. Although some efforts has been made in this direction (Muellbauer, 1974; Kakwani, 1977), there is a need to refine this approach in order to throw new lights on the relative preferences of household members in empirical investigations (Brown and Deaton, 1972).

The second issue is related to the problem of estimating the consumer unit scales. The fact that there is an identification problem in the model was acknowledged by Prais and Houthakker, further discussed by Forsyth (1960) and by Cramer (1969). Cramer (1969) says that "no amount of information about observed Engel curves will render (the specific household scales) determinate." He provides no general demonstration. Also Cramer (1973) and Muellbauer (1975) have argued that the system of weights is underidentified and that extraneous information is necessary to estimate the consumer unit scales. For this reason, Prais and Houthakker (1955), Nicholson (1949) and others have imposed arbitrary restrictions

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on their model. This paper will show that the consumer unit scae are not necessarily underidentified and that it is possible to estimate the unit scales without extraneous information.

This paper is organized as follows. Section II develops a theoretical model from utility maximization. The traditional approach is reviewed in Section III and compared with the theoretical approach. Using these results, it is shown in Section V that the consumer unit scales are identifiable without extraneous information. Finally, Section VI presents some concluding results.

II. Theoretical Model

The consumption patterns of a household is typically affected by several factors that may be economic or non-economic in nature. For example, besides prices and the income of the household, the occupations of its members, their age-sex distribution, their marital status, their race, etc., may be significant determinants of the consumption pattern. The house-household composition problem is primarily concerned with questions about how much does each household member, or the addition or deletion of a hold member, contrbiute to the cost of living of the household. According to Muellbauer (1980) and Somermeyer, earlier approaches designed to assess the impacts of household compostion were primarily based upon: (1) physiological and/or nutritional considerations of household members, or (2) the haphazard interaction of vested interest pressure group politics and administrative conventions, or (3) empirical investigations of household expenditure behavior. As Somermeyer points out, most of these approaches were lacking of a sound underlying theory. In their pionearing work, Prais and Houthakker attempted to overcome this limitation by use of the concept of equivalence scales. Equivalence scales are index numbers designed to indicate the relative contribution that household members of different ages and sex adds to the household a cost living or their expenditures on a food group. As such, household equivalence scales may be used to deflate the household income or expenditure and thus convert them to a standadized needs-corrected basis (Muellbauer, 1980) Here, let us consider the members of the household classified into different categories. This classification may be based on age, race, marital status, and/or any other criterion that appears to influence consumption behavior, and is used to define the consumer unit scales that incorporate household composition in Engel curve analysis. A "specific scale" originally introduced by Syndenstricker and King(1921) is the basis of our model. It gives to household members in each category different weights for different commodities. Accordingly, a weighted household size for each commodity is defined as a twice differentiable function of the family composition.

$$m_i = m_i\,(n_1, \ldots, n_g) \qquad i = 1, \ldots, r \tag{1}$$

where m_i is the weighted household size for the ith commodity, n_j is the number of members of the jth category in the household, $(n_j > 0)$, r is the number of commodities consumed by the household and g is the number of categories of persons.

Following Brown and Deaton (1972), the household utility function is defined as a twice differentiable function of X_i $(i = 1, 2, \ldots, r)$

$$U = f\,(X_1, \ldots, X_r) \tag{2}$$

where $X_i = \frac{q_i}{m_i}$ and q_i is the consumption level of the ith commodity purchased by the household. The utility function (2) is therefore a function of the consumption level per "standardized" person.

The household has a current income y and faces the budget constraint

$$y = \sum_{j=1}^{r} p_j q_j \tag{3}$$

where p_j is the price of the j-th commodity. The budget constraint can be alternatively written

$$y = \sum_{j=1}^{r} R_j X_j \tag{4}$$

where $R_j = p_j m_j$

Maximization of the utility function (2) subject to the budget constraint (4) allow us to write a set of r demand equations in terms of income and prices. The imposition of the homogeneity condition yields the demand functions

$$q_i = m_i D_i \left(\frac{p_1 m_1}{y}, \ldots, \frac{p_r m_r}{y}\right) \qquad i = 1, \ldots, r \tag{5}$$

With cross data where prices are assumed fixed, equation (5) becomes

$$q_i = m_i F_i \left(\frac{m_1}{y}, \ldots, \frac{m_r}{y}\right) \qquad i = 1, \ldots, r \tag{6}$$

Expressions (5) and (6) are the most general functions derived from utility maximization. Differentiating (5) with respect to n_k yields (Muell bauer, 1974).[1]

$$\frac{\partial q_i}{\partial n_k} = \frac{\partial m_i}{\partial n_k}\,\frac{q_i}{m_i} + m_i \sum_j \frac{\partial D_i}{\partial m_j} \cdot \frac{\partial m_j}{\partial n_k}, \; i = 1, \ldots, r \tag{7}$$

But

$$\frac{\partial D_i}{\partial m_j} = \frac{\partial D_i}{\partial p_j} \cdot \frac{p_j}{m_j} = \frac{\partial q_i}{\partial p_j} \cdot \frac{y_j}{m_i m_j} \tag{8}$$

[1] See J. Nuellbauer (1974) or appendix A for the justification of this statement.

Substituting (8) into (7) gives

$$\frac{\partial q_i}{\partial n_k} = \frac{\partial m_i}{\partial n_k} \cdot \frac{q_i}{m_i} + \sum_j \frac{\partial q_i}{\partial p_j} \cdot \frac{p_j}{m_j} \cdot \frac{\partial m_j}{\partial n_k} \qquad (9)$$

$$i = 1, \ldots, r$$

To put expression (9) in elasticity terms, multiply through by $\frac{n_k}{q_i}$

$$Eq_in_k = Em_in_k + \sum_j Eq_ip_j\, Em_jn_k, \qquad i = 1, \ldots, r \qquad (10)$$

where $E_{zy} = \frac{\partial z}{\partial y} \cdot \frac{y}{z}$ is the elasticity of z with respect to y. Expression (10) shows that a change in one household characteristic n_k has a direct and indirect effect on the demand for the ith good. The direct effect, Em_in_k is the effect through the household parameter m_i. The indirect effect, which is like a substitution effect, works through the interactions of the changes the which are precisely analogous to price changes and hence depends on the cross-elasticities between good i and the other goods. Also, expression (10) shows that estimate of Ep_in_k and Ep_ip_j, obtained from cross-section/times series data, are sufficient to identify the scales (i=1, . . . , r; k=1, . . . , g). Indeed (10) provides ($r \times g$) equations which, in the absence of lie ar dependency, can be solved for the ($r \times g$) unknown elements of the scales.

For fixed family composition and variable prices, the Slusky equation in elasticity form is

$$Eq_ip_j = Eq_ip_j | u - Eq_iy \cdot W_j \qquad (11)$$

where $(Eq_ip_j | u)$ is the substitution effect, $(Eq_iy \cdot W_j)$ the income effect and $W_j = \frac{p_jq_j}{y}$ is the budget share.

Substituting (11) into (10) yields

$$Eq_in_k = Em_in_k + \sum_j (Eq_ip_i | u \cdot Em_jn_k) - Eq_iy \sum_j (W_jEm_jn_k) \quad (12)$$

Expression (12) can show some lights on the sign of Em_in_k. Traditionally Em_in_k has been assumed to be positive. For example, Singh and Nagar (1973) have estimated their model under the constraint that $0 < \frac{\partial m_i}{\partial n_k} <$ 1. However the non-negativity of Em_jn_k appears to be an unjustified restriction. For instance, for Leontieff type indifference curves, we have $Eq_ip_j | u$ 0. Also, if the ith commodity has an arbitrarily small budget share and a negative income elasticity ($Eq_iy < 0$), and if $Em_jn_k > 0$ for $j \neq i$, it follows that $Eq_iy \sum_j (W_jEm_jn_k) < 0$. In this case, it implies that $Eq_in_k > Em_in_k$. As Eq_in_k is not a priori restricted in sign, it implies that Em_in_k may become negative under certain circumstances.[2]

[2] This is confirmed by the empirical results obtained by Singh and Nagar (1973).

As stated above, expression (10) shows that a change in household composition has a direct and indirect effect on the demand for the ith commodity. The direct effect measured the impact on the induced change in y on the consumption of the ith commodity. From (10) it can be expressed as

$$(1 + Eq_i p_i) \cdot Em_i n_k \tag{13}$$

For $Em_i n_k > 0$, the direct effect is positive when the ith good has an elastic demand ($Eq_i p_i < -1$) and is negative when the ith commodity has an inelastic demand ($Eq_i p_i > -1$). Opposite results are obtained for $Em_i n_k < 0$.

The indirect effect measures the impact of the induced change in the weighted household size for other goods m ($j=1, \ldots, r$; $j=i$) on the consumption of the ith commodity. From (10), it can be expressed as

$$\sum_{j \neq i}^{r} Eq_i p_j \cdot Em_j n_k \tag{14}$$

For $Em_j n_k > 0$, the jth commodity has a positive impact on the indirect effect when the ith and jth commodities are gross substitutes $(Eq_i p_j) > 0$ and has a negative impact when they are gross complements ($Eq_i p_j <$ 0). Again, opposite results are obtained for $Em_j n_k < 0$.

Thus in the case were $Em_j n_k > 0$ for all j commodities, the total effect $Eq_i n_k$ is positive when the ith commodity has an elastic demand and is a gross substitute with other goods. It is negative when the ith commodity has an inelastic demand and is a gross complement with other goods. For other situations, the total effect may be either positive or negative dependening on the relative magnitude of the terms involved in (10).

III. Traditional Model

The demand function (6) derived from utility maximization differs from the function usually used in empirical work (Prais and Houthakker, 1955; Cramer, 1973; Singh and Nagar, 1973) which is

$$\frac{q_i}{m_i} = g_i\left[\frac{y}{m_0}\right] \text{ or } q_i = m_i \cdot g_i\left[\frac{y}{m_0}\right] \tag{15}$$

where $m_0 = m_0 (n_1, \ldots, n_g)$ is the general weighted household size, a function of the family composition. Muellbauer (1974 p.116) has shown that $m.$ can be interpreted as a cost of living index given that the demand function (15) is derived from utility maximization. If m_i and m_0 are assumed linear functions of $(n_1, \ldots, n_g)$, the $\frac{\partial m_i}{\partial n_k}$ defines specific unit scales and $\frac{\partial m_0}{\partial n_k}$ the general unit scale (Prais and Houthakker, 1955).

Differentiating (15) with respect to n_k yields

$$\frac{\partial q_i}{\partial n_k} = \frac{\partial m_i}{\partial n_k} \cdot \frac{q_i}{m_i} + m_i \frac{\partial g_i}{\partial m_0} \cdot \frac{\partial m_0}{\partial n_k} \tag{16}$$

As $$\frac{\partial g_i}{\partial m_0} = \frac{\partial g_i}{\partial y} \cdot \frac{y}{m_0} = - \frac{\partial g_i}{\partial y} \cdot \frac{y}{m_i m_0} \tag{17}$$

Substituting (17) into (16), we can put it in elasticity form, which is called a specification relationship.

$$Eq_i n_k = Em_i n_k - Eq_i \cdot Em_0 n_k \tag{18}$$

Also, differentiating the budget constraint (3) with respect to y and n_k yields, respectively

$$\sum_i \frac{\partial q_i}{\partial y} p_i = 1 \tag{19}$$

and

$$\sum_i \frac{\partial q_i}{\partial n_k} p_i = 0 \tag{20}$$

or, in elasticity form

$$\sum_i Eq_i y \cdot W_i = 1 \tag{21}$$

and

$$\sum_i Eq_i n_k \cdot W_i = 0 \tag{22}$$

Substituting (19) and (21) into (22) yields

$$Em_0 n_k = \sum_i (W_i \cdot Em_i n_k) \tag{23}$$

Equation (23) states that the income scale ($Em_0 n_k$) is a weighted average of the specific scales ($Em_i n_k$) where the weights are the budget shares.

Substituting (23) into (18) yields

$$Eq_1 n_k = Em_i n_k - Eq_i y \sum_i (W_i \, Em_j n_k) \tag{24}$$

Comparing (24) with (12) shows that the traditional approach is equivalent to the utility maximization approach if and only if

$$\sum_j (Em_j n_k \cdot Eq_i p_j | u) = 0, \qquad i = 1, \ldots, r \tag{25}$$

As $Em_j n_k$ is either negative or positive and $Eq_i p_j \leq 0$, condition (25) may be satisfied for a number of utility functions. A sufficient condition for (25) is that the indifference curves are of the Leontief type with

$$Eq_i p_j | u = 0, \qquad i, j = 1, \ldots, r \tag{26}$$

Expression (26) implies that all goods are consumed in fixed proportions for a given family composition. However, expression (26) is not a necessary condition for (25) to be satisfied if Em_jn_k is allowed to become negative. This fact can expand the interpretation of Muellbauer (1974, p. 107)[3] Expression (24) shows the direct and indirect effect of a change in household composition on the demand for the ith commodity, in the traditional approach. The direct effect, measuring the impact of the induced change in m_i on the consumption of the ith commodity, is

$$(1 - w_i Eq_i y)\ Em_i n_k \tag{27}$$

For $Em_in_k > 0$, the direct effect is positive if the ith good inferior ($Eq_iy < 0$). However, for a superior good ($Eq_iy > 0$), the direct effect may be either positive for $Eq_iy > \frac{1}{w_i}$ or negative for $Eq_iy < \frac{1}{w_i}$ |depending on the magnitude of the income elasticity compared to the inverse of the budget share. It follows that the direct effect can be negative only if the income elasticity is greater than one. Opposite results are obtained for $Em_in_k < 0$.

The indirect effect, measuring the impact of induced change in m_j ($j = 1, \ldots, r$; $i \neq j$) on the consumption of the i-th commodity is:

$$- Eq_i y \cdot \sum_{j \neq i}^{r} (W_j \cdot Em_j n_k) \tag{28}$$

When $Em_jn_k > 0$ for all j commodities, the indirect effect is positive if the ith good is inferior ($Eq_iy < 0$) and negative if it is a superior good ($Eq_iy > 0$). Again, opposite results are obtained from $Em_in_k < 0$.

Again, in the case where $Em_jn_k > 0$, $j = 1, \ldots, r$, the negative if its income elasticity is greater than 1. In other situations, the total effect may be either positive or negative depending on the relative magnitude of the terms involved in (24).

IV. Derived Relationships

Consider the traditional model (15) commonly found in the literature. A number of relationship can be derived from this model. They include specification, aggregation, and symmetry relationships. The specification and aggregation relationships have already appeared in the literature (Cramer, 1973; Barten, 1964; Muellbauer, 1975). However, the symmetry relationship introduced in this paper appears particularly important because of its potential applications in empirical work.

The specification relationship has been derived in section III by differentiating (15) with respect to y and n_k. It correspond, in elasticity

[3] Mullbauer (1974) assumed that $\sum_j (Em_jn_k \cdot Eq_ip_j \mid u) = 0$, but it is too stronger restriction.

form, to expression (18).

The aggregation relationships have also been obtained in section III by differentiating the budget constraint with respect to y and n_k respectively. In elasticity form, they correspond to expressions (21) and (22).

Now, consider the symmetry condition. First, from (16) and (17), we have

$$\frac{\partial q_i}{\partial n_k}=\frac{\partial m_i}{\partial n_k}\frac{q_i}{m_i}-\frac{\partial q_i}{\partial y}\frac{\partial m_0}{\partial n_k}\frac{y}{m_0} \tag{29}$$

Differentiating (29) with respect to y yields

$$\frac{\partial^2 q_i}{\partial n_k \partial y}=\frac{\partial q_i}{\partial y}\cdot\frac{1}{m_i}\cdot\frac{\partial m_i}{\partial n_k}-\frac{\partial^2 q_i}{\partial y^2}\frac{\partial m_0}{\partial n_k}\frac{y}{m_0}-\frac{\partial q_i}{\partial y}\frac{\partial m_0}{\partial n_k}\frac{1}{m_0} \tag{30}$$

Multiplying (30) by p_i and summing over all commodities, we obtain

$$\sum_i\left(p_i\frac{\partial^2 q_i}{\partial n_k\partial y}\right)=\sum_i\left(\frac{\partial q_i}{\partial y}\frac{p_i}{m_i}\frac{\partial m_i}{\partial n_k}\right)-\frac{\partial m_0}{\partial n_k}\frac{y}{m_0}\sum_i\left(p_i\frac{\partial^2 q_i}{\partial y^2}\right)-\frac{\partial m_0}{\partial n_k}\frac{1}{m_0}\sum_i\left(p_i\frac{\partial q_i}{\partial y}\right) \tag{31}$$

As $\sum_i\left(p_i\frac{\partial^2 q_i}{\partial n_k\partial y}\right)=0$ and $\sum_i\left(p_i\frac{\partial^2 q_i}{\partial y^2}\right)=0$ from (19), and using (19), expression (31) becomes

$$\frac{\partial m_0}{\partial n_k}\cdot\frac{1}{m_0}=\sum_i\left(\frac{\partial q_i}{\partial y}\frac{p_i}{m_i}\frac{\partial m_i}{\partial n_k}\right) \tag{32}$$

This relationship is a necessary condition for the traditional model. Now, let us show that it is also a sufficient condition for symmetry overall commodities.

Differentiating (29) with respect to n_j yields

$$\frac{\partial^2 q_i}{\partial n_k\partial n_j}=\frac{\partial^2 m_i}{\partial n_k\partial n_j}\frac{q_i}{m_i}+\frac{\partial m_i}{\partial n_k}\frac{\partial q_i}{\partial n_j}\frac{1}{m_i}-\frac{\partial m_i}{\partial n_k}\frac{\partial m_i}{\partial n_j}\frac{q_i}{m_i^2}-\frac{\partial^2 q_i}{\partial y\partial n_j}\frac{\partial m_0}{\partial n_k}\frac{y}{m_0}-\frac{\partial q_i}{\partial y}\frac{\partial^2 m_0}{\partial n_k\partial n_j}\frac{y}{m_0}+\frac{\partial q_i}{\partial y}\frac{\partial m_0}{\partial n_k}\frac{\partial m_0}{\partial n_j}\frac{y}{m_0^2}$$

After substituting (29) into (33), we obtain

$$\frac{\partial^2 q_i}{\partial n_k\partial n_j}=\frac{\partial^2 m_i}{\partial n_k\partial n_j}\frac{q_i}{m_i}-\frac{\partial m_i}{\partial n_k}\frac{\partial q_i}{\partial y}\frac{\partial m_0}{\partial n_j}\frac{y}{m_0 m_i}-\frac{\partial^2 q_i}{\partial y\partial n_j}\frac{\partial m_0}{\partial n_k}\frac{y}{m_0}-\frac{\partial q_i}{\partial y}\frac{\partial^2 m_0}{\partial n_k\partial n_j}\frac{y}{m_0}+\frac{\partial q_i}{\partial y}\frac{\partial m_0}{\partial n_k}\frac{\partial m_0}{\partial n_j}\frac{y}{m_0^2} \tag{34}$$

The symmetry of (34) with respect to n_k and n_j implies that the expression is,

$$\frac{\partial m_i}{\partial n_k}\frac{\partial q_i}{\partial y}\frac{\partial m_0}{\partial n_j}\frac{y}{m_0 m_i}+\frac{\partial^2 q_i}{\partial y \partial n_j}\frac{\partial m_0}{\partial n_k}\frac{y}{m_0} \tag{35}$$

The expression (35) is also symmetric with respect to n_k and n_j that is:

$$\frac{\partial m_i}{\partial n_k}\frac{\partial q_i}{\partial y}\frac{\partial m_0}{\partial n_j}\frac{1}{m_i}+\frac{q_i}{\partial y \partial n_j}\frac{\partial m_0}{\partial n_k} \tag{76}$$

$$=\frac{\partial m_i}{\partial n_j}\frac{\partial q_i}{\partial y}\frac{\partial m_0}{\partial n_k}\frac{1}{m_i}+\frac{\partial^2 q_i}{\partial y \partial n_k}\frac{\partial m_0}{\partial n_j} \tag{36}$$

Multiplying (36) by p, summing overall commodities, and using $\sum_i p_1\left(\frac{\partial^2 q_i}{\partial y \partial n_j}\right)=0$, we obtain,

$$\frac{\partial m_0}{\partial n_j}\sum_i\left(\frac{\partial m_i}{\partial n_k}\frac{\partial q_i}{\partial y}\frac{p_i}{m_i}\right)=\frac{\partial m_0}{\partial n_k}\sum_i\left(\frac{\partial m_i}{\partial n_j}\frac{\partial q_i}{\partial y}\frac{p_i}{m_i}\right) \tag{37}$$

However, note that the relation (37) is satisfied if (32) holds. Therefore, (32) is a sufficient condition for symmetry over all commodities. This symmetry condition, expressed in elasticity form, becomes

$$Em_0 n_k=\sum_{1=i}^{r}(W_i\, Eq_i y \cdot Em_i n_k) \tag{38}$$

This relationship is now shown to be crucial in the identification of the consumer scales.

V. Identification of the Consumer Unit Scales

From the previous sections, four type of relationship have been derived: the specification condition (18), the aggregation conditions (21) and (22), and the symmetry condition (38). These relationships involve $Eq_i n_k$, $Eq_i y$, $Em_0 n_k$, $Em_j n_k$, and W_i. In principle, $Eq_i n_k$, $Eq_i y$, and W_i are all observable and directly estimable. But $Em_i n_k$ and $Em_0 n_k$ are not directly estimable and must be obtained from the relationships just derived.[4]

Previous work (Cramer, 1973; Muellbauer, 1975) utilized the specification condition (18) and the two aggregation conditions (21) and (22). As done in section III, combining them yields expression (24), which gives a set of (rg) equations with (rg) unknown specific consumer unit scales. However, Muellbauer (1975) has shown that the system of equations (24) cannot be solved because of linear dependency. This should bo obvious given the way the problem is set. First, note that the two ag-

[4] Note that the unit scales are expressed in elasticity form, i.e., they are defined up to a factor of proportionality. In empirical work, a normalization rule is usually imposed on this scales. When the adult male is chosen as the basis for normalization, it produces "adult equivalent scales", commonly found in the literature.

gregation conditions (21) and (22) do not provide any information to help identify the unit scales. In the this approach, the specification condition (18) is the only one that provides information on the consumer unit scales. It gives (rg) linear equations. As there are $[(r+1)g]$ unknown weights, g constituting the income scale and (rg) constituting the special scales, it shows clearly that is impossible to solve the system of equation for the unit scales in this context. However, that does not mean the unit scales are underidentified. It only means that there are (g) missing independent equations that can be generated within the system. These (g) equations can be indeed be obtained from the symmetry condition (38).

More specifically, the unit scales, consist·ng of $[(r+1)g]$ unknowns, can be obtained from the specification condition (18) that provides (rg) equations, and from the symmetry condition (38) that provides (g) equations. In the absence of linear dependency, solving the system of equations yields estimates of the unit scales. Thus, substituting (38) into (18) gives

$$Eq_i n_k = Em_i n_k - Eq_i y\,[\sum_{i=1}^{r} (Eq_i y \cdot Em_i n_k \cdot W_i)] \tag{39}$$

Expression (39) provides (rg) equations that can be solved to produce (rg) estimates of the specific scales. In a second stage, estimates of the income scale can be obtained either from the specification condition (18) or the symmetry condition (38).

These results show that the use of the symmetry condition makes the consumer unit scales identifiable. This corrects a mistake commonly made in the literature concerning the underidentification of the unit scal es.

VI. Conclusion

This paper has reviewed the interpretation of the traditional consumer unit scale hypothesis in the context of utility maximization. The traditional approach exhibits some attractive characteristics. It can be derived from utility maximization while remaining fairly simple in its model specification. Consumer unit scale research is likely to be useful in commodity demand analysis. Unit scale values will provide a means of understanding crossectional expenditure variation due to socio-demographic differences among households. They will also give a basis for forecasting family expenditures on specific commodities as socio-demographic characteristics of the population change. Agricultural economists may thus find equivalence scales valuable in predicting food consumption levels several years into future, based on the expected demographic profile of the population at some future date. The main problem has been to avoid the underidentification of the consumer unit scales, which has greatly limited research on this topic. This paper has shown that in the context of the traditional approach, the unit scales are identificable without extraneous information. This should give a good basis for applied work to investigate the influence of socio-economic factors on household consumption.

The traditional approach appears well suited for cross-section data analysis, without price variations. Future research is clearly needs to investigate alternative models incorporating both prices and family composition in the demand functions.

APPENDIX A

The direct utility function would be written

$$U = f(q_1, \ldots, q_r) \qquad s.t. \quad y = \sum_j p_j q_j$$

Based on Barten model, it could be defined by

$$U = f\left(\frac{q_i}{m_i}, \ldots, \frac{q_r}{m_r}\right) \qquad s.t. \quad y = \sum_j p_j \frac{q_j}{m_j}$$

That is,

$$U = f(x_1, \ldots, x_r) \qquad s.t. \quad y = \sum_i R_j X_j$$

where $\frac{q_i}{m_i} = X_i \qquad R_j = p_j m_j$

The demand function derived from the above equation.

$$X_i = D_i\,(R_1, \ldots R_r, y) \tag{A}$$

Imposing the symmetry condition to equation (A)

$$X_i = D_i\left(\frac{R_1}{y}, \ldots, \frac{R_r}{y}\right) \qquad \text{or}$$

$$\frac{q_i}{m_i} = D_i\left(\frac{p_1 m_1}{y}, \ldots, \frac{p_r m_r}{y}\right) \tag{B}$$

Differentiation of (B) with respect to n_k gives,

$$\frac{\partial q_i}{\partial n_k} = \frac{\partial m_i}{\partial n_k}\frac{q_i}{m_i} + m_i \sum_j \left(\frac{\partial D_i}{\partial\left(\frac{\partial p_j m_j}{y}\right)}\frac{p_j}{y}\frac{\partial m_j}{\partial n_k}\right) \tag{C}$$

Note $$\frac{\partial q_i}{\partial p_j} = m_i \frac{\partial D_i}{\partial\left(\frac{p_j m_j}{y}\right)}\frac{m_j}{y}$$

Thus $$\frac{\partial D_i}{\partial\left(\frac{p_j m_j}{y}\right)} = \frac{\partial q_i}{\partial p_j}\frac{y}{m_0 m_j} \tag{D}$$

Substitution (D) into (C) yields

$$\frac{\partial q_i}{\partial n_k} = \frac{\partial m_i}{\partial n_k}\frac{q_i}{m_i} + m_i \sum_j \frac{\partial q_i}{\partial p_j}\frac{y}{m_i m_j}\frac{p_j}{y}\frac{\partial m_j}{\partial n_k}$$
$$= \frac{\partial m_i}{\partial n_k}\frac{q_i}{m_i} + \sum_j \frac{\partial q_i}{\partial p_j}\frac{p_j}{m_j}\frac{\partial m_j}{\partial n_k}$$

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