

## A QUADRATIC PROGRAMMING MODEL FOR THE DETERMINATION OF OUTPUT PRICE SUPPORT AND INPUT PRICE SUBSIDY

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The purpose of this paper is to show how a mathematical programming model can be formulated to examine problems associated with a few important price policy alternatives i.e., allocation of limited funds for output price support and input price subsidy. A comprehensive survey of the problems includes Krishna (1967). The methodology delineated here is quite different from Barker and Hayami (1976) for the similar problems; they applied a neoclassical demand-supply model.

This paper is organized as follows: (1) illustrates how quadratic terms are to be shown up in the model when output price support and/or input price subsidy considered, (2) explains multiperiod extension of the model areas of further applications and problems.

### 1. From LP to a Quadratic Programming Model

A LP model in matrix notation is written as

$$\begin{aligned} &\max C'X \quad (C \text{ and } X: n \times 1) \\ &\text{st. } AX \leq B \quad (A: n \times n; B: n \times 1) \\ &\quad X \geq 0 \end{aligned}$$

A minimal need for the explanation of notation is as follows<sup>1</sup>:

- $x_1$ : Acreage of the crop in question the government plans to provide for output price support and input subsidy (hectare)
- $c_1$ : Net revenue coefficient for  $x_1$  (gross sale value of output minus cash expenses from one hectare).
- $a_{11}$ : Land coefficient.
- $a_{21}$ : Cash expenses per hectare for  $x_1$  (or capital coefficient).
- $b_1$ : Land availability in hectare (for a representative farm).
- $b_2$ : Capital availability (for a representative farm).
- $\alpha$ : Percentage rate of price increase (over the market price of the product

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<sup>1</sup> Subscripts are to be attached to identify individual entries: 1, 2, ...,  $n$  for  $C, X$  and  $B$ ;  $a_{11}, a_{12}, \dots, a_{1n}$  and etc. for  $A$  (e. g.  $a_{11}$  represents coefficient in the first column and row in  $A$ ).

to be sought).

$\beta$ : Percentage of the total cash expenses to be provided as input subsidy (unknown).

With  $\alpha\%$  price support and  $\beta\%$  input subsidy, the increased net revenue  $c'_1$  is expressed as

$$c'_1 = c_1 \left(1 + \frac{\alpha}{100}\right) - a_{21} \left(1 - \frac{\beta}{100}\right), \quad \alpha \geq 0, \quad \beta \geq 0$$

With  $\beta\%$  input subsidy saving in total cash expenses is  $\frac{a_{21}}{100}\beta$ .

Thus, with  $\alpha$  and  $\beta$  considered, the model is written as:

$$\begin{aligned} \max & \left[ c_1 \left(1 + \frac{\alpha}{100}\right) - a_{21} \left(1 - \frac{\beta}{100}\right) \right] x_1 + c_2 x_2 + \dots \\ \text{st. } & a_{11}x_1 + a_{12}x_2 + \dots \leq b_1 \dots (1) \\ & a_{21} \left(1 - \frac{\beta}{100}\right) x_1 + a_{22}x_2 + \dots \leq b_2 \dots (2) \\ & \dots \dots \dots \\ & \dots \dots \dots \\ & \dots \dots \dots \\ & \frac{c_1}{100} \alpha + \frac{a_{21}}{100} \beta = k \dots (n)' \end{aligned}$$

(n)' represents an equation needed for the fund constraint for a particular farm offered by government.

As shown above the level of  $x_1$  is to be raised with  $\alpha$  and  $\beta$ , other things held constant. However, the government budget (considered per farm basis) constitutes a new constraint for the problem (The sum of  $k$  for all farms in an enterprise is equal to government budget. Further discussion in this regard leads to the well-known aggregation bias problem in a programming model).

## 2. Multiperiod Extension of the Model

The quadratic programming model presented above can be formulated as a multistage or multiperiod programming problem.

The model captures some important and realistic features in agricultural production. The most important aspect of all is that the model takes explicit account of the time required for production adjustment, which is assumed away in the static analysis. This approach is particularly useful for the study of perennial crop production adjustments, since the model explicitly takes account of the changes in input requirements and expected yields over time. The model can also incorporate other aspects which are expected to develop over time and are considered to influence producers' decision for the crop. They include expected changes in input-output relationships and capital availability over time. Various recursive relationships can also be built into the model. For example, the net revenue generated at the end of the first crop year (period 1)  $c'_1$  would contribute

to relaxing the capital constraint in period 2 [For a detailed explanation for the compilation of MPS file for the similar problem, see Yoo (1985)]. As such, the model would be useful in analyzing the farm growth process (under government price support policy). Other areas of application include an economic analysis of capital investment (for R and D for a commodity) and technical development (if the relationship is established in manageable order of functional relationship).

General problems encountered in applying mathematical programming model in empirical research (such as time, manpower, highly qualified team and etc.) have been numerous mentioned in the literature. A few specific ones in the construction of a multiperiod model include:

a) The planning horizon of the model needs to be specified. The problem arises because fruit trees (if introduced in the model as competing crops in resource use with others), for example, have different net revenue streams over time, and they need to be compared on a comparable length of time. A closely related problem is determining the length of unit production decision period over the planning horizon. In most of the perennial production response studies, one year may be considered long enough to use as a unit period. However, for farm products harvested more than once a year, the length of period may need to be adjusted appropriately.

b) The choice of proper discount rate has become a very real and difficult problem. In order to use the interest rate to discount the future revenue streams, the capital market should be at least approximately perfect, where the interest rate clears capital markets. However, estimating (or identifying) the market-clearing interest rate may not be easy (even for a short-term period); the problem is reflected in different interest rates coexistent at any given time. With common capital rationing, the cost of capital may also vary according to the amount of the funds to be borrowed. Often the discount rate applicable for the project may be higher than the interest charged by financial institutions. Another related problem is that the interest rate is subject to change from time to time (This would cause a problem when the necessary funds need to be procured over time.).

c) The matrix dimension of the model for a study increases with the extended period of time under consideration, leading to computational complexities for solutions. However, with rapid development of large capacity computers, it has become possible to handle the large-side model without great difficulty encountered in the past. Specific computing resources for nonlinear programming problems include Bruce and Saunders (1977 and 1983).

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