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OPTIMAL REGIONAL INDUSTRY MIX UNDER UNCERTAINTY : A MOTAD APPROACH

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I . Introduction

State and local governments are frequently interested in attracting industries to promote employment and economic growth. At the same time, they want to minimize regional unemployment and economic instability that may result from the industries. Thus, policy makers may encourage certain industries to grow while discouraging others, inducing moderate alteration in the industrial structures. To achieve this, information about the trade-off between economic growth and economic instability in a region is essential for the policy makers to choose an optimal set of trade-offs.

Conroy(1973) suggested a portfolio theory approach. He incorporated Siegel's(1966) measure of regional economic instability in cross-sectional models where portfolio variance and other measures of industrial diversification were used as independent variables. Barth(1974) applied a portfolio approach to the investigation of the relationship between industrial mix and employment stability of a region. This approach permits the specification of the conditions under which a new or expanding industry will reduce fluctuations in regional employment. Conroy(1975) found that regional variations in rates of fluctuation and variation in industrial diversification were significant, indicating trade-off between regional economic instability and industrial diversification. Louis(1980) developed a model to measure regional industrial diversification, in a Markowitz portfolio context, using the notion of a regional efficiency frontier. He argued that a region can be considered to be optimally diversified when it is on this efficiency frontier. Kort(1981) developed a model of regional economic instability and industrial diversification with emphasis on city size. Brewer (1984) presented empirical findings about trade-off between regional economic instability and industrial diversification with the use of portfolio approach.

Most literatures confirm that there exists trade-off between industrial diversification and regional economic stabilization in terms of employment. The approach employed in those literatures is mostly portfolio approach.

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The major weakness of the portfolio approach used in the regional analysis is that it does not incorporate resource constraints of the region concerned. Each region has its own unique resource constraints and preference of certain industries. The inclusion of the resource constraints and reflection of regional preference for the certain characteristics of industries in the model would improve substantially practical usefulness of efficient sets for the regional decision makers.

The objective of this paper is to develop a new approach that addresses the weakness of the portfolio approach in the analysis of regional economic stabilizations.

This paper first presents a theoretical concept of the new approach and suggests a model. Hypothetical data and analysis results follow the model. Hypothetical data were used to demonstrate the application of the new approach. Finally, summary and conclusions are provided.

Ⅱ. Theoretical Framework

We assume regional policy makers want to maximize their utilities by maximizing economic growth as well as minimizing economic instability in the region, given resource constraints. We further assume that the policy makers are averse to economic instability and that the utility function of policy makers are precisely the reflection of the regional residents' desires. We use employment level and its variations as proxy for economic growth and instability as in the case with portfolio approach. Then, the objective of the region is to maximize regional employment (expected value) as well as to minimize employment instability (risk). This objective the policy makers face could be achieved by using risk programming, where expected value is maximized and risk is minimized at the same time. The MOTAD(minimization of total absolute deviation) model that was suggested by Hazell(1971) is one of mathematical programming models that has been widely used in deriving out efficient sets of farm enterprise mix under risk and uncertainty. The MOTAD approach is an approximation to mean-variance(E-V) efficiency approach whose efficient set is a subset of the second stochastic dominance(SSD) efficient set. The MOTD approach would be more powerful than portfolio approach used in industrial mix analysis because it considers resource constraints. Also MOTAD approach would be better than portfolio approach since the linear programming codes required to solve MOTAD formulations are more widely available, better understood and more dependable than the quadratic programming codes required to implement the portfolio approach.

Ⅲ. Model Formulation

We regard each industry produces employments. We assume that we can measure the standard unit industry which requires minimum number of employees with certain amount of resources. Other important assumptions are to be no change of technology over time, constant cost industry, independence among industries in business activities, and constant output price and production cost per unit output over years.

Then, an initial LP model that does not consider risk(employment instability) would be constructed as follows:

Maximize $\sum_{j=1}^{n} C_j X_j$

Subject to

$\sum_{j=1}^{n} a_{ij} X_j \leq b_i$ for all i resources

$X_j \geq 0$

where

$C_j =$ average number of employment of standard unit industry over years to unpaid resources per unit the *j-th* industrial activity,

$X_j =$ level of the *j-th* industrial activity,

$b_i =$ the amount of *i-th* unpaid resources, and

$a_{ij} =$ the amount of the *i-th* resource required per standard unit industry of the *j-th* industrial activity(techincal coefficients).

In the rows b_i includes water, land, labor upper bound, input market index, output market index, infrastructure index, environmental index, public service index, tax index, and others that may constrain industrial activities. We also need to have standard unit industry in the row.

In the columns X_j is the industrial activities, that is, the amount of standard unit of *j-th* industry. C_j shows the average number of employment per standard unit of *j-th* industry over years. The LP solution is the maximum value of parameter λ to be used in the MOTAD model shown below.

Now, the MOTAD model is constructed. The MOTAD model used in this analysis assumes a utility function.

$$U(Z) = a + bZ + c\,[\,Z - E(Z)\,]$$

where

a, b, and c are positive constants and

Z is the random variable.

The form of the model is:

Minimize Ld^-

Subject to

$$AX \leq B$$

$$DX + Id^- \geq 0$$

$$C'X = \lambda$$

and

$$X,\ d^-,\ \lambda \geq 0$$

where

$L =$ a 1 by S vector where S is the number of years considered

$d^- =$ a S by 1 vector of yearly negative employment deviation from mean employment which is the mean of employment series

$A =$ a m by n matrix of technical coefficients, where m is the number of constraints and n is the number of industrial activities

$X =$ a n by 1 vector of industrial activity levels

$B =$ a m by 1 vector of resource level or constraints

$D =$ a S by n matrix of employment deviation

$I =$ a S by S identity matrix

$C' =$ a 1 by n vector of expected employment

$\lambda =$ a scaler used to represent the employment constraint.

In the MOTAD model, we minimize Ld^- which represents the summed total negative deviations over all years, subject to those constraints above. The efficient frontier is developed by parameterizing λ from zero to its maximum value. The maximum λ value is obtained from the solution of the initial LP model that did not consider risk or uncertainty. The tradeoff occurs between expected value(expected employment) and risk(negative employment deviation). In this MOTAD model, risk is measured as linear deviations from the mean employment. Implicitly, risk is undesirable, and hence is minimized.

Data required for MOTD analysis are 1) historical average number of employment per standard unit for each industry, 2) resource and /or preference constraints, and 3)resource requirement per standard unit for each industry.

Ⅳ. Hypothetical Data and Analysis

Employment data over ten years are assumed for six industries as shown in Table 1. Each industry shows variability of employment for standard unit over ten years. Industry 16 shows high employment with low coefficient of variation while industry 14 and 12 show low employment with high coefficient of variation. Mean employment per standard unit for each industry is high for industry 15 and 16, and low with industry 12 and 14. Correlations among industries are not high as shown in Table 2.

With the initial LP model that does not consider risk(employment instability) we want to maximize number of employment given resource constraint as shown in Appendix 1. In the rows of the initial tableau four resource constraints and one standard unit constraint are assumed. In the c_j columns yearly mean employments for *j-th* standard unit industry are included. The final tableau shows the optimum solution, that is, 159 standard units of industry 11 and 113 standard units of industry 13. This mix of industry gives 12,563 persons of employment in this region. The sum of industry unit solution and slacks of standard unit constraints is equal to the total amount of standard unit constraints, which is consistent with the model.

With this optimal employment solution initial MOTAD tableau was constructed as shown in Appendix 2. This MOTAD tableau is consistent with MOTAD model. It consists of resource constraints R, standard unit constraint UC, technical coefficients matrix, matrix of negative deviation from the mean employment, a row of mean employment, transfer matrix, and maximum λ value. The maximum λ value is the one that has been generated from initial LP tableau. The optimal solution generated from MOTAD model with risk consideration is shown in the solution tableau in

TABLE 1 **Number of Employment for Selected Industries in "S" State(1976~1985) for Standard Unit**

Year	11	12	13	14	15	16
1970	48	30	68	12	34	60
1977	29	58	21	22	81	90
1978	40	18	34	25	90	65
1979	49	5	57	9	88	62
1980	60	44	74	26	95	85
1981	39	8	13	5	40	70
1982	50	13	46	18	67	63
1983	52	7	49	3	20	40
1984	54	33	56	26	45	70
1985	38	16	45	20	19	80
Mean	45.9	23.1	46.3	16.8	57.9	68.5
STD	8.7	16.8	18.3	8.5	28.2	13.6
C.V.	0.2	0.7	0.4	0.5	0.5	0.2

TABLE 2 **Correlation Coefficients of Selected Enterprise Employment**

	11	12	13	14	15	16
11	1.00000	−0.14323	0.81450	0.05201	0.04832	−0.36584
12		1.00000	0.09198	0.65397	0.34614	0.70404
13			1.00000	0.21137	0.05198	−0.17040
14				1.00000	0.49069	0.67224
15					1.00000	0.40306
16						1.00000

the same Appendix 2. Industries 11 and 13 are selected to produce employment of 12,563 persons with the risk value of 14,181.

When λ is parameterized from the maximum employment 12,563 to the lower employment value with 500 intervals optimum industry mixes are generated, giving different levels of employment and risk(negative employment deviation) as shown in Appendix 3. Some selected tradeoffs between expected employment and risk, and associated optimum industry mixes are shown in Table 3. As risk increases industries 11 and 13 are preferable to get higher employment. Industry 16 and industry 11 are selected at medium employment and risk level. Industries 14 never enters the industry mix. This is due to its low mean employment and high employment variation. At the highest employment and risk level only industry 11 is selected.

These trade-offs generate efficient frontier(E–A) as shown in Figure 1. The frontier shows four distinct range of trade-offs between expected employment and risk. At region A an additional increase of certain number of employment cost smaller risk increase than any other regions, B, C, and D. At region D smaller additional increase of employment needs to take high increase of risk. In the different set of trade-offs corresponding optimal industry mixes are suggested. This provides regional decision makers with different alternative choices of industrial mixes given regional resource and preference constraints.

TABLE 3 **Trade-off between Expected Employment and Risk, and Associated Industry Mixes**

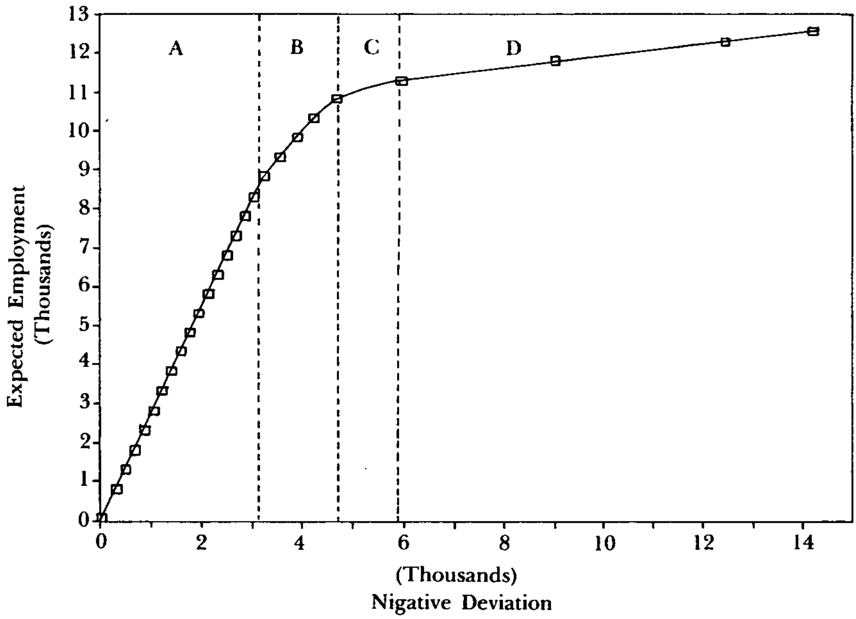
Expected Employment	Negative Deviation	Industry Mixes					
		11	12	13	14	15	16
1813	666	18	0	0	0	0	13
4313	1586	43	0	0	0	0	33
6813	2506	68	0	0	0	1	53
9313	3573	104	0	0	0	0	65
10813	4701	147	4	0	0	0	57
11813	9001	157	0	55	0	0	29
12313	12455	158	0	94	0	0	9
12563	14181	159	0	113	0	0	0

V. Summary and Conclusions

Portfolio approach has been frequently used to find optimum industry mix in a region in the past. One of the major weakness of portfolio approach as is used in regional analysis is that it does not consider resource constraints and regional preference, and that it uses quadratic programming which is more complicated than linear programming algorithm. As a better alternative MOTAD approach was suggested.

The MOTAD efficient frontier is widely recognized as a decision aid in

FIGURE 1 **Efficient Frontier**



selecting efficient crop mixes under risk and uncertainty. The application of MOTAD approach to hypothetical data for a region generated optimal industry mixes successfully. The efficient frontier demonstrated the trade-off between expected employment and employment instability (risk) at all levels of risk employment.

Some major limitations and improvements in the analysis deserve to be mentioned. First, the MOTAD result may be significantly influenced by the degree of measurement precision of the standard unit industry. Second, business cycles among industries were not incorporated in the model. A different method of calculating mean employment rather than whole period average may be necessary. Third, careful studies for development of locational preference index of industries may be required so that the practical usefulnes of the MOTAD model could be substantially improved.

Although the analysis with hypothetical data is simplified one, the MOTAD approach with some improvement for optimal industry mixes in the regional analysis would be quite useful as a good decision aid to regional policy makers.

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APPEDIX 1 **Initial Tableau**

Reglon	Maximize	B	11	12	13	14	15	16
C		(RMS)	45.9	23.1	46.3	16.8	57.9	68.5
UC	L	5000	1	1	1	1	1	1
R1	L	1000	2	4	6	8	10	12
R2	L	500	2.5	0.7	0.9	1.1	1.5	1.9
R3	L	800	0.3	1.2	0.5	1.5	1.6	0.6
R4	L	600	0.6	0.8	1.3	0.2	0.5	0.9

Solution
Optimal
Function Value : 12363.63

				12	14	15	16	R1	R2
				23.1	16.8	57.9	68.5	0	0
Returns		Type	Level	real	real	real	real	slack	slack
45.9	11	real	159.0909	0.045454	-0.04545		0.045454	-0.06818	0.454545
46.3	13	real	113.6363	0.651515	1.348484	1.666666	1.984848	0.189393	-0.15151
0	UC	slack	472.7272	0.303030	-0.30303	-0.66666	1.03030	-0.12121	-0.30303
0	R3	slack	695.4545	0.860606	0.839393	0.766666	-0.40606	-0.07424	-0.06060
0	R4	slack	356.8181	-0.07424	-1.52575	-1.66666	-1.70757	-0.20530	-0.07575
Z				33.25151	60.34848	77.16666	93.98484	5.639393	13.84848
Shadow	Price			9.151515	43.54848	19.26666	25.48484	5,639393	13.84848

APPENDIX 2

MOTAD C	Minimize	B (RNS)	I1 0	I2 0	I3 0	I4 0	I5 0	I6 0	Y1 1	Y2 1	Y3 1	Y4 1	Y5 1	Y6 1	Y7 1	Y8 1	Y9 1	Y10 1
UC	L	5000	1	1	1	1	1	1										
R1	L	1000	2	4	6	8	10	12										
R2	L	500	2.5	0.7	0.9	1.1	1.5	1.9	technical coefficient									
R3	L	800	0.3	1.2	0.5	1.5	1.6	0.6										
R4	L	600	0.6	0.8	1.3	0.2	0.5	0.9										
T1	G	0	2.1	6.9	21.7	-4.8	-23.9	-8.5	1									
T2	G	0	-16.9	34.9	-25.3	5.2	23.1	21.5		1								
T3	G	0	-5.9	-5.1	-12.3	8.2	32.1	-3.5			1							
T4	G	0	3.1	-18.1	10.7	-7.8	30.1	-6.5				1						
T5	G	0	14.1	20.9	27.7	-11.2	37.1	16.5	negative deviation				1					
T6	G	0	-6.9	-15.1	-33.3	-11.8	-17.9	1.5						1				
T7	G	0	4.1	-11.1	-0.3	1.2	9.1	-5.5							1			
T8	G	0	6.1	-16.1	2.7	-13.8	-37.9	-28.5								1		
T9	G	0	8.1	9.9	9.7	9.2	-12.9	1.5									1	
T10	G	0	-7.9	-7.1	-1.3	3.2	-38.9	11.5										
AVGM	E	12563	45.9	23.1	46.3	16.8	57.9	68.5	mean employment									1

Solution
Optimal
Functlon Value: 14181.96

Cost	Name	Type	Level	I2 0 real	I4 0 real	I5 0 real	Y1 1 real	Y4 1 real	Y5 1 real	Y7 1 real	Y8 1 real	Y9 1 real	R1 0 slack	R2 0 slack	T2 0 slack	T3 0 slack	T6 0 slack	T10 0 slack	AVGF 1OEMO slack
0	I1	real	159.0897	0.029131	-0.12312	-0.03436							-0.07824	0.429845					0.001783
0	I3	real	113.5868	-0.06123	-2.04322	0.166111							-0.24982	-1.23008					0.077883
0	I6	real	0.024970	0.359096	1.708799	0.756004							0.221284	0.543400					-0.03923
1	Y2	real	5561.826	26.12247	-85.3135	10.46777							-12.4003	-35.5398	-1				2.844233
1	Y3	real	2335.834	-4.42449	-11.6772	36.58644							-2.75992	-10.6920		-1			0.831153
1	Y6	real	4880.122	-17.4768	-83.2520	-13.7395							-9.19084	-38.8109			-1		2.664684
1	Y10	real	1404.184	-11.0790	-20.0800	-47.6495							-3.48763	-4.45243				-1	0.566587
0	UC	slack	4727.298	0.673008	1.457550	0.112247							0.106777	0.256837					-0.04042
0	R3	slack	695.4646	1.006420	1.533269	1.073650							0.015612	0.160047					-0.01593
0	R4	slack	356.8608	0.538941	1.392146	-0.37573							0.172556	0.852140					-0.06700
0	T1	slack	2798.709	-11.2199	-54.3212	21.00642	-1						-7.46634	-30.4090					2.027348
0	T4	slack	1708.394	15.20095	-25.5513	-33.3431		-1					-4.35398	-15.3614					1.093935
0	T5	slack	5389.932	-16.2604	-41.3381	-20.5091			-1				-4.37205	-19.0463					1.535077
0	T7	slack	618.0546	9.262782	-10.4902	-13.4497				-1			-1.46290	-0.85731					0.199762
0	T8	slack	1276.420	5.878124	-41.1685	16.59274					-1		-7.45838	-16.1860					1.339476
0	T9	slack	2390.456	-9.71938	-27.4533	15.36694						-1	-2.72508	-7.63495					0.711058

APPENDIX 3 **Trade-off between Expected Employment and Risk, and Associated Industry Mixes**

Expected Employment	Negative Deviation	Industry Mixes					
		11	12	13	14	15	16
63	23	0	0	0	0	0	0
813	299	8	0	0	0	0	6
1313	483	13	0	0	0	0	10
1813	666	18	0	0	0	0	13
2313	850	23	0	0	0	0	17
2813	1034	28	0	0	0	0	21
3313	1218	33	0	0	0	0	25
3813	1402	38	0	0	0	0	29
4313	1586	43	0	0	0	0	33
4812	1770	48	0	0	0	1	36
5313	1954	53	0	0	0	1	40
5813	2138	58	0	0	0	1	44
6313	2322	63	0	0	0	1	48
6813	2506	68	0	0	0	1	52
7313	2690	73	0	0	0	1	56
7813	2874	78	0	0	0	1	59
8313	3058	83	0	0	0	1	63
8813	3256	86	0	0	0	1	67
9313	3573	104	0	0	0	0	65
9813	3924	119	0	0	0	0	63
10313	4291	133	1	0	0	0	60
10813	4701	147	4	0	0	0	57
11313	5946	156	0	16	0	0	45
11813	9001	157	0	55	0	0	29
12313	12455	158	0	94	0	0	9
12563	14181	159	0	113	0	0	0