Journal of Rural Development 14 (1991) : 79 ~ 90

THE FACTOR BIAS OF TECHNICAL CHANGE UNDER PRODUCTION UNCERTAINTY

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ABSTRACT

The impact of production uncertainty on factor-augmenting technical change is examined by using and Itô stochastic control model. The results indicate that thechnical progress will be biased toward risk-reducing inputs and against risk-increasing inputs. This bias may reinforce or counteract the bias caused by relative input price movements.

Hicks [8] proposed that changes in the relative prices of inputs would act as a spur to inventions that economized the use of the factor that had become relatively more expensive. This intuitive proposition is now called the induced innovation theory. The theory has been formulated in microeconomic terms and tested empirically [1; 4; 5; 6; 9; 11; 12; 15; 18; 19; 20; 21].

The purpose of this paper is to show how the incorporation of production uncertainty alters Hicks's proposition. The inputs that increase the variance of output are separated from those that reduce the variance of output. The results show that, under production uncertainty, firms will deviate from the technology development and adoption path implied by Hicks's proposition so as to economize the use of inputs that increase the variance of production. This divergence will depend on the level of production uncertainty and on the

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degree of risk aversion of the decision maker. The results are derived by Itô stochastic control.[1] The results depend on the specified assumptions; consequently, the results may not have immediate use in policy analysis. Nevertheless, the results are meaningful in that they may help to explain some of the deviations from Hicks's proposition that have been found in the empirical studies listed previously. Should it be possible to validate the theoretical results that follow, the concepts would help researchers develop innovations with a greater likelihood of acceptance in a target audience.

In the next section, the stochastic control model is presented. This model contains endogenous factor-augmenting technical change and allows the derivation of some propositions regarding the impact of production uncertainty on factor-augmenting technical change. The results are presented in terms of the implied deviations from the induced innovation theory. The final section contains concluding remarks and implications for policy.

I. The Model

A production function that explicitly incorporates factor-augmenting technical change may be written as

$$Y(t) = F[A(t)\,X_1(t),\,B(t)\,X_2(t)],$$

where $F[.]$ is assumed to be a differentiable, quasi-concave function, $X_1(t)$ and $X_2(t)$ are the levels of two inputs at time t, and $A(t)$ and $B(t)$ reperesent the levels of factor augmentation at time t [19]. It may be useful to consider X_1 to be an imput that increases production uncertainty (e.g., fertilizer or yield increasing varieties) and X_2 to be an input that decreases production uncertainty (e.g., irrigation or disease-resistant varieties).

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[1] Refer to Arnold [2], Hertzler [7], and Malliaris and Brock [13, 80−118] for a detailed discussion of Itô differentiation and dynamic programming.

It is assumed that a representative firm can fully internalize the benefits from its research and that it finances all research internally. This assumption limits the analysis to firms that can obtain patents or to government-sponsored research programs that conduct research on problems specific to the domestic environment. A less formal interpretation would allow us to view the research that ensues as that which would be desired by firms that ultimately use the proceeds of the research. If these firms signal researchers via market forces or the political process, then research programs should reflect the desires of these end users in a manner consistent with the assumption.

The levels of research expenditures E_1 and E_2 are assumed to determine the rate of increase in $A(t)$ and $B(t)$, as shown in the technical progress functions:

(1) $dA = Ah_1(E_1)dt$, and
$dB = Bh_2(E_2)dt$

[20, 60–69]. The function $h_1(.)$ and $h_2(.)$ are assumed to be continuous and twice differentiable, with positive first and negative second derivatives.

Assume that the firm maximizes the expected discounted utility of the ternimal state variables wealth W and the value of the stocks of accumulated knowledge A and B such that

(2) $J(S_0) = \text{Max } E[e^{-rt}V(S_T) \mid S_0 = \bar{s_0}]$

where J is the indirect utility function resulting from the optimization; S is the vector of state variables W, A, and B; r is the firm's rate of time preference; T is the terminal time; and $V(.)$ represents the utility function of terminal state variables.

Excluding the value of accumulated knowledge, the firm's wealth consists of an inventory of risk-free assets, L, valued at market price, P_L (Where negative L denotes liabilities), or $W = P_L L$. Using Itô's lemma, the change in wealth can be specified as

(3) $dW = LdP_L + P_L dL + dP_L dL$

[10;13]. The term LdP_L represents capital gains on the current inventory of assets whereas the remaining two right-hand variables are the net value of additions to wealth from sources other than capitial gains. In the absence of consumption, these last two terms represent the income or losses from production such that

$$(4) \qquad (P_L + dP_L)dL = \pi dt + d\pi,$$

where $\pi dt + d\pi$ is the income or losses from production and $P_L + dP_L$ is the price of risk-free assets, dL assumed to be purchased with this production income [14].

If we specify the change in the value of risk-free assets as

$$\frac{dP_L}{P_L} = \delta_L dt,$$

where δ_L is the known growth rate of P_L, by substituting this equation and (4), we can rewrite (3) as

$$(5) \qquad dW = (\delta_L W + \pi)dt + d\pi.$$

We can introduce distributions of production level and input prices as follows:

$$dY = Y\sigma_y dz_y,$$
$$dP_1 = P_1\delta_1 dt, \text{ and}$$
$$dP_2 = P_2\delta_2 dt,$$

where σ_y is the standard deviation of the percentage change in production, δ_i is the forecast of the growth rate for input price i, and z_y is a Wiener process in which $E[dz_y(t)] = 0$ and $E[dz_y(t)]^2 = dt$ [10]. If input i increases, the first-derivative of the standard deviation of output with respect to input X_i, $\partial\sigma_y/\partial X_i > 0$, then, X_i, is risk increasing. The opposite is true for risk-reducing technologies [7].[2]

Expected production income at the current time may be expressed as

$$\pi = PY - P_1X_1 - P_2X_2 - E_1 - E_2,$$

where P is the known output price. By using Itô's lemma, we can stochastically differentiate for the change in production income to get

$$\begin{aligned} d\pi &= PdY - X_1dP_1 - X_2dP_2 \\ &= PY\sigma_y dz_y - X_1P_1\delta_1 dt - X_2P_2\delta_2 dt. \end{aligned}$$

By substituting these expressions into (5), we get the following dynamic budget constraint

$$(6) \qquad \begin{aligned} dW = [&\delta_L W + PY - P_1X_1(1 + \delta_1) - P_2X_2(1 + \delta_2) \\ &- E_1 - E_2]dt + PY\sigma_y dz_y. \end{aligned}$$

In summary, the objective of the firm is to maximize the expected discounted utility of terminal stock of state variables (2) subject to the dynamic budget constraint (6) and the technical progress constraints (1). The variables X_1, X_2, E_1, and E_2 are the control and choice variables, whereas W, A, and B are the stock and state variables.

The Itô version of the Bellman equation associated with this model specification is

$$\begin{aligned} -J_t = \text{Max}[&J_w\{\delta_L W + PF - P_1X_1(1 + \delta_1) - P_2X_2(1 + \delta_2) \\ &- E_1 - E_2\} + J_A\{Ab_1(E_1)\} + J_B\{Bb_2(E_2)\} + 1/2\ J_{ww} \\ &\{P^2F^2\sigma_y^2\}], \end{aligned}$$

where J_w and J_{ww}, respectively, are the first- and second-order derivatives with respect to wealth, and J_A and J_B are the first-order derivatives with respect to A ant B, respectively.

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[2] The static uncertainty literature contains some other definitions of risk-reducing and risk-increasing inputs. See Pope and Kramer[16] for a detailed discussion.

The expression to be maximized is a dynamic certainty equivalent denominated in utils. For convenience, we call it a stochastic Hamiltonian [13]. The stochastic Hamiltonian and first-order conditions for X_1, X_2, E_1 and E_2 can be written as

$$H = J_w \; [\delta_L W + PF(AX_1, \; BX_2) - P_1X_1(1 + \delta_1) - P_2X_2 (1 + \delta_2) - E_1 - E_2] + J_A[Ab_1(E_1)] + J_B[Bb_2(E_2)] + 1/2 J_{ww} P^2 F^2 \sigma_y^2,$$

(7) $$\frac{\partial H}{\partial X_1} = J_w[PF_1A - P_1(1 + \delta_1)] + J_{ww}[P^2\sigma_y^2 FF_1A + P^2F^2 \sigma_y \sigma_{yx1}] = 0,$$

(8) $$\frac{\partial H}{\partial X_2} = J_w[PF_2B - P_2(1 + \delta_1)] + J_{ww}[P^2\sigma_y^2 FF_2B + P^2F^2 \sigma_y \sigma_{yx2}] = 0,$$

(9) $$\frac{\partial H}{\partial E_1} = -J_w + J_A A b_1' = 0, \text{ and}$$

(10) $$\frac{\partial H}{\partial E_2} = -J_w + J_B B b_2' = 0.$$

F_1 and F_2 are the first-order derivatves of the production function with respect to AX_1 and BX_2, respectively, b_1' and b_2' are the first-order derivatives of the technical progress function with respect to E_1 and E_2, respectively, and σ_{yx1} and σ_{yx2} are the first derivation of standard deviation with respect to inputs X_1 and X_2, respectively.

The first-order conditions in (7) and (8), input decision rules under production uncertainty, can be depicted as

$$PF_1A = P_1(1 + \delta_1) - J_{ww}/J_w \; [P^2\sigma_y^2 \; FF_1A + P^2F^2\sigma_y\sigma_{yx1}], \text{ and}$$
$$PF_2B = P_1(1 + \delta_2) - J_{ww}/J_w \; [P^2\sigma_y^2 \; FF_2B + P^2F^2\sigma_y\sigma_{yx2}].$$

These relationships have an intuitive interpretation. In the deterministic case with no uncertainty, $\sigma_y = 0$, the firm applies inputs to the point at which the marginal value product equals the expected input price. Because J denotes the indirect utility function relating from optimization, $-J_{ww}/J_w$ will be the Arrow-Pratt measure of absolute risk aversion [3;17]. For a risk-neutral firm in which J is a linear function of wealth ($J_{ww} = 0$), the result is the same as for the deterministic case. For a ristaverse firm, for which J_{ww}/J_w is negative and marginal risk premium is positive, inputs will be utilized to the point at which their marginal value product equals input price plus a marginal risk premium. If an input is risk reducing, the marginal risk premium will decrease. On the other hand, if an input is risk increasing, the marginal risk premium will increase. Therefore, if X_i is risk reducing, a risk-averse firm will apply more X_is than will a risk-neutral firm. On the contrary, if X_i is risk increasing, a risk-averse firm will apply less X_is than will a risk-neutral firm.

Combining first-order conditions (9) and (10) yields

$$(11) \qquad J_w = J_A A b_1'(E_1) = J_B B b_2'(E_2).$$

This equation implies that E_1 and E_2 will be determined at the point the marginal utility of wealth equals the marginal utility of A or B times the stock level of factor augmentation and marginal productivity of the technical progress function. To retrieve the relationship between production uncertainty and the bias of technical change, we differentiate (11) partially with respect to W, A and B, and also differentiate (11) by using Itô's lemma:

$$(12) \qquad J_{ww} = J_{Aw} A b_1' = J_{Bw} B b_2', \text{ and}$$

$$(13) \qquad A b_1' dJ + J_A b_1' dA + b_1' dJ_A dA = B b_2' dJ_B + J_B b_2' dB + b_2' dJ_B dB.$$

The evolution of the marginal indirect utility of state, that is another group of first-order conditions [13, 111-112], can be expressed as

$$(14) \qquad dJ_w = -J_w \delta_L dt + J_{ww} P F^* \sigma_y dz_y$$

$$(15) \qquad dJ_A = -[J_w P F_1^* X_1^* + J_A b_1^* + J_{ww} P^2 F^* F_1^* X_1^* \sigma_y^2] dt + J_{Aw} P F^* \sigma_y dz_y, \text{ and}$$

$$(16) \qquad dJ_B = -[J_w P F_2^* X_2^* + J_B b_2^* + J_{ww} P^2 F^* F_2^* X_2^* \sigma_y^2] dt + J_{Bw} P F^* \sigma_y dz_y,$$

where * represents the optimal value. Substituting (11), (12), (15), and (16) into (13) and simplifying yields.

(17) $$\frac{b_1'(E_1^*)}{b_2'(E_2^*)} = \frac{PF_2^* X_2^* B + J_{ww}/J_w P^2 F^* F_2^* X_2^* \sigma_y^2 B}{PF_1^* X_1^* A + J_{ww}/J_w P^2 F^* F_1^* X_1^* \sigma_y^2 A}.$$

This set of equations constitutes the model.

II. The Results

The framework described in the preceding in based on the existing literature. The derivation of the propositions that are the principal contribution of this paper is now straightforward. Substitute (7) and (8) into (17) to obtain

(18) $$\frac{b_1'(E_1^*)}{b_2'(E_2^*)} = \frac{X^*_2[P_2(1 + \delta_2) - J_{ww}/J_w P^2F^{2*}\sigma_y\sigma_{yx2}]}{X^*_1[P_1(1 + \delta_1) - J_{ww}/J_w P^2F^{2*}\sigma_y\sigma_{yx1}]}.$$

Equation (18) summarizes the important results of this paper. Assume, without loss of generality, that $b_1(E_1)$ and $b_2(E_2)$ have the same functional form. Then, if $b_1(E_1^*)/b_2(E_2^*)$ is less (equal or greater) than 1, the rate of factor augmentation for X_1 is higher (equal or lower) than that for X_2, i.e., $\dot{A}/A > (= \text{or} <) \dot{B}/B$, where $\dot{A}$ and $\dot{B}$ are the time rates of change. The term J_{ww}/J_w will be negative for the individual's concave utility functions. The term $P_i(1 + \delta_i)$ represents the expected costs of input i caused by an expected increase in its price. The term J_{ww}/J_w $(P^2F^{2*}\sigma_y\sigma_{yx_i})$ will be negative for as long as the i-th input increases the variability of output.

If there is no uncertainty, σ_y will equal zero and δ_1 and δ_2 will be the known input prices. To further simplify, assume that both inputs have similar costs, $P_1X_1 = P_2X_2^* = P_iX_i^*$. This simplifies (18) to

(19) $$\frac{b_1'(E_1^*)}{b_2'(E_2^*)} = \frac{P_iX_i^*(1 + \delta_2)}{P_iX_i^*(1 + \delta_1)}.$$

Equation (18) and (19) leads directly to the following propositions.

Proposition 1 (Hicksian). When there is no uncertainty or when firms are risk neutral, relative input prices will determine the rate of bias of endogenous technical progress.[3] This Hicksian proposition is, however, a special case. In situations in which firms are risk averse and the production process is uncertain, the bias of technical change will be different from that predicted by Hicks. The magnitude of this difference will depend on the difference between σ_{yx1} and σ_{yx2} and on σ_y and J_{ww}/J_w.

If X_1 is risk increasing ($\sigma_{yx1} > 0$) and X_2 is risk reducing ($\sigma_{yx2} < 0$), then technical change will be biased toward X_2 in a manner that is independent of relative input prices. This result can best be seen by assuming that $\delta_1 = \delta_2 = \delta_i$; i.e., the expected growth rates of input prices are similar, and if this is the case, (18) can be written as

$$(20) \qquad \frac{b_1'(E_1^*)}{b_2'(E_2^*)} = \frac{P_i X_i^*(1+\delta_i) - J_{ww}/J_w \; X_2^* P^2 F^{2*} \sigma_y \sigma_{yx2}}{P_i X_i^*(1+\delta_i) - J_{ww}/J_w \; X_1^* P^2 F^{2*} \sigma_y \sigma_{yx1}}.$$

This equation leads directly to proposition 2.

Proposition 2. For a risk-averse firm facing production uncertainty, the rate of endogenous technical progress will be biased even if there is no difference in the growth rates of input prices. Given risk aversion, the rate of factor-augmenting technical change for those inputs whose use increases production risk is higher than the rate of factor-augmenting technical change for those inputs whose use decreases production risk.

The increased production risk is an implicit cost associated with the use of the risk increasing input, while the decreased production risk is an implicit benefit associated with the use of the risk reducing input. The firm seeks to economize, through higher factor-augmenting technical change, on the use

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[3] Sato and Ramachandran [19] define the rate of bias of endogenous technical progress as the difference in the rates of change of factor augmentation and show that in a steady state this will equal the difference in the rates of growth of input prices. Using their definition, we call this result the Hicksian proposition.

of inputs with higher explicit and implicit relative costs.

The risk-induced bias may reinforce or counteract the bias caused by relative input price movements. This result implies that $\dot{A}/A$ can be equal to $\dot{B}/B$, even if the firm faces different expected growth rates in input prices. This implication leads to corollary 1.

Corollary 1. For a risk-averse firm facing different expected growth rates in input prices and production uncertainty, the Hicks-neutral technical change can not be excluded because of the existence of production uncertainty and risk.

III. Concluding Remarks

The theory of endogenous technical progress assumes that changes in the relative prices of inputs will lead to the invention of technologies. Such inventions may be undertaken by a public institution or by individual firms. In this paper, a dynamic, microeconomic, factor-augmenting technical change model is developed to investigate how production uncertainty affects the bias of technical change. Uncertainty has been modeled as a set of Wiener processes in whcih the level of variance of these processes implies the level of uncertainty. We show that uncertainty in prices or production may affect the rate of bias of technical progress.

Under production uncertainty, technical progress for the risk-averse firm will be affected by the characteristics of inputs (i.e., risk increasing or risk reducing). Technical change will be biased toward risk-reducing inputs and against risk-increasing inputs. It is also shown that the Hicksian proposition is a special case in which there is no uncertainty or in which firms are risk neutral.

From a policy perspective, the principal conclusion from this analysis is that the degree of output uncertainty will influence the types of technology that are developed and adopted. For example, new hybrid varieties of corn might be more favorably received in parts of the United States where weather conditions are more stable, whereas risk-reducing innovations would be more useful in areas where weather patterns are more volatile. Internationally, one would expect the risk-increasing technologies of the Green Revolution to be successfully adopted in countries or regions with stable output patterns. Also, the results suggest that research should concentrate on technologies that reduce risk in regions where production uncertainty is high.

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