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AN ECONOMETRIC ANALYSIS OF THE KOREAN MEAT DEMAND SYSTEM: USING BAYESIAN MONTE CARLO METHODS

SHIN YONG-DO*

I. Introduction

The Korean economy experienced a rapid growth over the last two decades. The rapid economic growth also led to fairly rapid rates of income growth and urbanization resulting in a rising demand for better quality foods such as meat and other livestock products.That is, as per-capita income and population have grown, especially in urban areas, changing dietary preferences have led to a relatively higher growth in the consumption of animal protein than in cereals and vegetables. Because domestic meat production, and of beef in particular, has expanded at a slower rate than demand, the Korean government has been importing beef since 1976, but restrictively.

However, under the Uruguay Round multilateral trade negotiations, the Korean government is required to open the domestic market wider for livestock products including beef, pork and chicken, through tariffication, quota increases and liberalization. In order to understand better the likely effect of possible effects on domestic meat markets by such an import policy, it would be useful to study the Korean meat demand system systematically

Demand theory suggests that any plausible demand system should satisfy four properties: (1) adding-up, (2) homogeneity, (3) symmetry, and (4) concavity. In addition to these restrictions, it seems reasonable to expect that no pair of foods playing essentially the same

* Research Fellow, KISDI, 1-33 Juam- Dong Kwachun, Kyunggi-do 427-070, Korea. I thank Gary Williams, and two anonymous referees for helpful comments. All remaining errors are, of course, my sole responsibility.

role in the diet should be complements. Therefore, the constraint that all meats are net substitutes can be viewed here as another requirement that any well-behaved meat demand system should satisfy.[1]

While equality constraints, such as homogeneity or symmetry, are easily imposed on the parameters of the demand system for estimation and hypothesis testing, inequality restrictions like concavity and net substitutability are not easily handled in conventional estimation approaches. Recently Geweke (1986) developed a new approach to impose inequality constraints on the regression models. Some works have used his method to impose curvature restrictions on a set of substitution elasticities obtained from the translog cost function (Chalfant and White, 1987) or net substitutability among meats for an AID system (Hayes, Wahl, and Williams, 1990).

In this paper, the Korean meat demand system is estimated and tested using a method based on Geweke's work. Specifically, the same procedure used in Chalfant and White, and Hayes et al is used to estimate the Korean demand system and test inequality constraints in the demand system. Because of the feedback effects among relevant meats, a demand system for beef, pork, chicken meat and marine products[2] is considered here.[3] The meat group is assumed to be weakly separable from other food groups as well as other commodities in this paper. Because of data limitations, beef from beef cattle and beef from dairy cattle as well as imported beef are considered to be the same good here.

There are many alternative specifications and functional forms for a demand system, including the translog, Rotterdam, Almost Ideal Demand System (AIDS) models, etc. However, because the AIDS model proposed by Deaton and Muellbauer (1980), has been considered to have many desirable properties,[4] the AIDS is used in this study for estimation and hypothesis testing in the Korean meat

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[1] Net Substitutability requires that the compensated cross-price elasticities must be greater than or equal to zero.

[2] Because fish data is not available in Korea, and because fish is the most important component of marine products, a marine product price index is used as a proxy for the price of fish

[3] Even though numerous food group items could be included in the demand system, the data limitations make this difficult and may be impractical.

[4] For details, see Deaton and Muellbauer (1980), and Blanciforti and Green (1983)

demand system.

This paper finds that the data set satisfies homogeneity and symmetry. The result substantially supports the concavity of the consumer's expenditure function underlying an almost ideal demand system. But the sample information is not consistent with the prior belief about net substitutability, for it reveals a very low probability that beef, port, chicken meat and marine products are net substitutes. There is some evidence of net complementarity between marine products and beef.

II. Empirical Model

Extending the model proposed by Working (1943) and Leser (1963, 1976), Deaton and Muellbauer suggested the Almost Ideal Demand System as a particular representation of price independent generalized logarithm (PIGLOG) preferences. Such preferences allow perfect aggregation over consumers. In addition, the functional form they chose is locally flexible in the sense that it can attain arbitrary values for elasticities of substitution at some point.

The AIDS in budget share form is as follows:

$$W_i = \alpha_i + \sum_j \gamma_{ij} \log P_j + \beta_i \log \frac{X}{P} \tag{1}$$

where W_i is the budgetary share allocated to the ith meat, P_j is the price of meat j, X denotes the per capita expenditure on all meats, and P is a price index defined by

$$\log P = \alpha_0 + \sum_k \alpha_k \log P_k + \frac{1}{2} \sum_k \sum_j \gamma_{kj} \log P_k \log P_j \tag{2}$$

In many practical cases where prices are relatively collinear, P is approximately proportional to some price index. Deaton and Muellbauer utilized Stone's price index, $\log P^* = \sum_k W_k \log P_k$, in that case. Because a high correlation is expected among price indexes in time series estimation, equation (1) is redefined as

$$W_i = \alpha_i + \sum_j \gamma_{ij} \log P_j + \beta_i \log \frac{X}{P^*} \quad (3)$$

Equation (3) is a good first order approximation for the complete AIDS model. Therefore, in this paper, the above Linear Approximate/Almost Ideal Demand System (LA/AIDS) model is used to estimate the parameters. Using simple formulas, elasticities can be computed from the estimated parameters of the LA/AIDS model.

The AIDS model can satisfy adding up, homogeneity and symmetry conditions by restricting the parameters of the system. First, if $\sum_{i=1}^{n} \alpha_i = 1$, $\sum_{i=1}^{n} \gamma_{ij} = 0$ and $\sum_{i=1}^{n} \beta_i = 0$, the sum of the budget shares is equal to one (adding-up). Second, if $\sum_{j=1}^{n} \gamma_{ij} = 0$, the estimated demand functions are homogeneous of degree zero in prices and expenditure(homogeneity). Finally, if $\gamma_{ij} = \gamma_{ji}$, the symmetry condition holds.

The above equality constraints on the parameters of the AIDS model can be easily imposed and estimated, but inequality constraints like concavity of the expenditure function and net substitutability among meats are not as easily handled. When inequality constraints are violated in one system, the conventional approach for imposing these is to try alternative specifications of functional forms until the inequality constraints are satisfied.[5] An alternative approach, proposed by Geweke, is to use the Bayesian approach for estimating and testing inequality constraints. Therefore, the Bayesian approach is used in this paper.

Bayes' theorem suggests that the posterior probability density function is proportional to the product of the prior probability density function (pdf) and the likelihood function, i.e.,

$$P(\theta|y) \propto P(\theta) \times L(y|\theta) \quad (4)$$

where $P(\theta|y)$ is the posterior pdf for the parameter vector θ given the sample information, $P(\theta)$ is the prior pdf for the parameter vector θ, and $L(y|\theta)$ is the likelihood function. To estimate the parameters in the AIDS model with restrictions, the posterior pdf and the specific

[5] This sequential pretesting procedure is well discussed in Judge and Brock (1978).

loss function are needed. It is easily known that if the loss function is quadratic, the optimal point estimate of θ is the mean vector of the posterior pdf. For the absolute error loss function, the median vector of the posterior pdf is the optimal point estimate (Zellner, 1971, pp. 24-25).

Let us suppose estimation of demand systems with restrictions. Equality constraints like homogeneity and symmetry can be easily imposed on the prior pdf of parameters. Because inequality constraints on parameters in the demand system are also imposed through the prior pdf, it is truncated. The truncated posterior pdf over the parameter space[6] is relevant to estimating the parameters in the system. When the quadratic loss function is assumed here, the problem is how to find the mean vector of the truncated posterior pdf. As Chalfant and White (1987) pointed out, it is not necessary to be able to solve the appropriate integral to find the mean vector of the posterior pdf, and even though it is possible, it cannot be done without extensive computations. Therefore, the Monte Carlo procedure is adopted in order to avoid such problem.

Following Chalfant and White, the AIDS model is estimated using the Bayesian Monte Carlo estimation technique.[7] Because the share equations in the AIDS model are related through the adding up constraint, the covariance matrix is singular. Therefore, after deleting one equation, only three share equations are estimated using the iterative seemingly unrelated regression (SUR) method.Parameters of the deleted equation are obtained by the adding up restriction. Iterative SUR provides the maximum likelihood estimates and these are invariant to the choice of equation for deletion (Chalfant, 1987).

The exact procedure for the Bayesian Monte Carlo estimation with restrictions is as follows. First, the parameter ($\hat{\theta}$) and the covariance matrix ($\hat{\Sigma}$) are estimated with homogeneity and symmetry restrictions. Second, based on the assumption that the error terms in the share equations are multivariate-normally distributed, the multivariate-normal distribution with mean vector $\hat{\theta}$ and the

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[6] This parameter space is not the complete parameter space but a restricted space which satisfies the equality constraints.

[7] See Kloek and Dijk (1978), and Stewart (1983) for more discussion about the Bayesian analysis using the Monte Carlo method.

covariance matrix $\hat{\Sigma}$ is used to generate random vectors by the Monte Carlo method. Third, each replication is checked to see if the inequality constraints (concavity and net substitutability) are satisfied. Finally, the sample mean vector for random vectors which satisfy inequality constraints is calculated. The sample mean vector is an estimate of the mean vector for the truncated posterior pdf.

Similarly, testing inequality restrictions is done using Monte Carlo integration. The proportion of Monte Carlo replications in the sample satisfying restrictions estimates the probability that the demand system meets restrictions. A very low probability would cast doubt on either the prior beliefs, or the functional form for the demand system.

III. Empirical Results

The Korean demand system is estimated using aggregate Korean data for the years 1962 to 1992, taken from various yearbooks published by National Livestock Cooperatives Federation and Bank of Korea. The marine product share equation in the demand system was chosen for deletion here. Especially, because retail prices are not available for the data period, the wholesale prices are used in the AIDS model.

First, symmetry was imposed on the demand system, and tested.[8] Symmetry was accepted at the 5% level of significance. Then, the demand system with homogeneity and symmetry imposed was estimated. Because estimates of elasticities are of interest, compensated price elasticities and elasticities of substitution were calculated at the mean values of observed shares, which are shown in Table 1 and 2.

Table 1 indicates that all own (compensated) price elasticities are negative as expected and that most of the cross price elasticities are positive. That is, most meats are substitutes for one another at the mean shares in Korea. The negative cross price elasticity for beef and marine products suggests that beef and marine products are

[8] In the LA/AIDS model, the imposition of symmetry automatically imposes homogeneity (McKenzie and Thomas, 1984). Therefore, the test of symmetry alone implies the test of both restrictions.

TABLE 1 Expenditure and Compensated Price Elasticities with Homogeneity and Symmetry Restrictions

	Beef	Pork	Chicken Meat	Marine Product
Beef	-0.4066	0.2039	0.2910	-0.0883
	(0.1517)	(0.2131)	(0.0989)	
Pork	0.1547	-0.5472	0.0640	0.3285
		(0.3690)	(0.1224)	
Chicken Meat	0.5929	0.1717	-0.8430	0.0782
			(0.3610)	
Marine Product	-0.0495	0.2422	0.0214	-0.2143
Expenditure	1.1308	0.8135	0.5293	1.1936
	(0.0634)	(0.0833)	(0.0959)	

Note : Numbers in parentheses are standard errors of elasticities.

TABLE 2 Elasticities of Substitution with Homogeneity and Symmetry Restrictions

	Beef	Pork	Chicken Meat	Marine Product
Beef	-1.8704	0.7115	2.7271	-0.2273
	(0.7035)	(0.7429)	(0.8799)	
Pork	0.7115	-1.9078	0.5990	0.8445
		(1.2945)	(1.1534)	
Chicken Meat	2.7271	.5990	-7.9012	0.2014
			(3.3685)	
Marine Product	-0.2273	0.8445	0.2014	-0.5506

Note : Numbers in parentheses are standard errors of elasticities.

complements for each other, which is contrary to a priori expectations. However, Table 2 suggests that the concavity restriction was satisfied with the estimated parameters

These findings do not mean that the concavity condition holds with 100% probability, nor is the probability that the net substitutability restriction holds zero. Bayesian analysis with Monte Carlo integration is still necessary for generating probability statements with respect to concavity and net substitutability.

For Monte Carlo integration with inequality restrictions, 20,000 random samples were drawn from the multivariate normal distribution, i.e., 12-variate normal distribution.[9] First, each replication was checked whether concavity was satisfied. Of 20,000 replications, 10,360 satisfied the concavity restriction, representing a probability of 51.8% that the restriction holds. Using the formula suggested by Geweke, the precision of proportion (standard error of proportion) for the concavity restriction was calculated, which was 0.00353 in this example with 20,000 replications.[10] This implies that the Korean meat demand system is consistent with the concavity restriction. Next, each replication was checked for the concavity and net substitutability restrictions. In our random sample, the percentage of replications that satisfied these restrictions was 7.2%, and a standard error of proportion was 0.0017. That is, out of 20,000 trials, 1,440 satisfied the concavity and net substitutability conditions. This evidence casts strong doubt on the prior belief that all meats should be substitutes. However, the compensated price elasticities with consistent random samples were calculated and are reported in Table 3. Table 4 also shows elasticities of substitution at the mean vector of replications, consistent with the inequality restrictions.

TABLE 3 Expenditure and Compensated Price Elasticities with All Restrictions

	Beef	Pork	Chicken Meat	Marine Product
Beef	-0.4919	0.1860	0.2210	0.0851
Pork	0.1408	-0.5673	0.1274	0.2899
Chicken Meat	0.4502	0.3421	-1.0688	0.2763
Marine Product	0.0477	0.2203	0.0759	-0.3437
Expenditure	1.1308	0.8135	0.5295	1.1936

[9] For random vector generating method, refer to IMSL STAT/LIBRARY, pp. 1033-1034.

[10] A standard error of proportion is calculated using the formula $\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$ where $\hat{P}$ is the sample proportion (consistent with inequality restrictions) and n is the total number of replication. For more greater precision, more Monte Carlo replications could be generated.

TABLE 4 Elasticities of Substitution with All Restrictions

	Beef	Pork	Chicken Meat	Marine Product
Beef	-2.2636	0.6481	2.0710	0.2190
Pork	0.6481	-1.9779	1.1932	0.7683
Chicken Meat	2.0710	1.1932	-10.0179	0.7103
Marine Product	0.2190	0.7683	0.7103	-0.8829

TABLE 5 Proportion of Replications Satisfying Partial Net Substitutability

Meat for which no inequality restrictions were imposed	Proportion of replications satisfying partial net substitutability
Beef	22.5%
Pork	10.4%
Chicken Meat	14.3%
Marine Product	35.7%

Following Hayes, Wahl and Williams, the meat that is primarily responsible for the low proportion in net substitute restriction is checked and reported in Table 5. This was done by removing one meat at a time from the net substitutability restriction and keeping the other restrictions. Table 5 suggests, for example, that 22.5% of the replications satisfies the net substitutability restriction among pork, chicken meat, and marine products. The percentage of replications is highest when marine product is not restricted to being a substitute of the other meats. The result also shows some evidence that beef and marine products are complements for each other. Provided that all the conditions used in the analysis are valid, this finding can be interpreted as showing an unique meat consumption pattern in Korea.

IV. Conclusion

The Almost Ideal Demand System (AIDS) was used to estimate the Korean meat demand system and test the restrictions which are

consistent with demand theory. It was found that the Korean data set is consistent with homogeneity and symmetry. While the equality restrictions (homogeneity and symmetry) can be easily imposed for estimation and hypothesis testing, inequality restrictions (concavity and net substitutability) cannot be done easily. Consequently, a Bayesian approach with Monte Carlo integration was used as an alternative.

Based on the prior belief of homogeneity and symmetry, the optimal estimates, satisfying the inequality conditions, were found using the Monte Carlo procedure with the truncated posterior probability density function. The results suggested that the concavity restriction was satisfied about 52% of the time with a standard error of 0.00353, and about 7% with a standard error of 0.0017 for concavity and net substitutability restrictions. This indicates that the sample information is consistent with concavity, but the prior belief about net substitutability is suspected. Finally, the analysis suggests that beef and marine products are complements to each other.

However, all such inferences in the Korean meat demand system are conditional on the particular data set used here, the functional form for the demand system, and the basic assumptions like separability, aggregation and stable preference.Therefore, the results shown in this paper should be interpreted carefully, and further research is recommended in this light.

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