A GARCH TEST OF STRUCTURAL CHANGE: IN CASE OF U.S. BEEF DEMAND

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I. Introduction

It is often questioned, in demand analysis among others, whether model structure (i.e., parameters) is stable over time (Chalfant and Alston, 1988; Khan, 1974). Model stability is suspected when economic or political shocks, which can be perceived as permanent, occur during the sample period or when an analysis gives unexpected results. Sources of such change in model structure would be introduction of new technology, shift in consumer's preference, or some institutional changes.

Past research on the U.S. beef demand found evidence that the parameters governing the beef consumption changed over time (Chavas, 1983; Eales and Unnevehr, 1988; Moschini and Meilke, 1989). The U.S. consumers use less and less red meat due to changes in preference, increasing health concerns, demographic shifts, and/or improved marketing service to consumers. Structural changes in the beef demand have also been detected in case of Korea (Bae, 1988; Koo, Yang, and Lee, 1992) and Australia (Martin and Porter, 1985).

In quantitative analysis, adequate modeling of structural change, if any, is important for correct inferences and improved predictions (Moschini and Meilke, 1989). A simple way to detect model instability is to include a time trend in the model (e.g., Eales and Unnevehr, 1988), but this approach is not that informative. Bae (1988) used a dummy variable to capture structural change in meat consumption in Korea. However, the method cannot model smooth transition in demand responsiveness.

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Brown, Durbin, and Evans (1975) developed the Cusum and Cusum of squares tests for parameter stability in the linear model. The Cusum test is designed to detect whether there is a gradual change in parameters and when it happens. On the other hand, the Cusum of squares test detects abrupt, rather than smooth, structural changes. Even though the power of the Cusum and the Cusum of squares test are questioned (Garbade, 1977), they are still widely used for the test after being extended by Dufour (1982).

The Cusum tests are based on k-step ahead recursive stability analysis. The null hypothesis is that the normally distributed k-step ahead recursive residuals have a zero mean for the Cusum test and have a constant variance for the Cusum of squares test. Thus, if a structural change occurs abruptly rather than systematically, the Cusum test cannot detect the change. On the other hand, if the change occurs gradually, the Cusum of squares test is likely to fail to reject the hypothesis. Furthermore, the Cusum tests are based on order statistics to get confidence intervals (semi-nonparametric). The power of nonparametric test is usually regarded as low.

Little effort has been made to overcome the weaknesses of the Cusum tests. Fortunately, those properties of recursive residuals permit use of the GARCH model developed by Engle (1982) and Bollerslev (1986), which uses past information and allows variances to change over time. Since the GARCH model estimates both mean and variance simultaneously, tests of both smooth and abrupt changes in model structure is possible at the same time. Since the test used the t-statistics, the power of test should be acceptable.

The objective of this paper is to apply the GARCH process to the model stability test using the recursive residuals. The performance of the GARCH test is compared to the conventional Cusum tests. The tests are conducted with the U.S. data for beef demand during 1970 to 1987.

This paper is organized as follows. The second section briefly reviews the Cusum tests and the method to derive the recursive residuals, to which the GARCH as well as the Cusum tests are applied. In the third section, the GARCH process is introduced for the test of structural change. The fourth section describes the data and estimation procedures. The results are presented and interpreted in the fifth section, followed by the sixth section that concludes this paper.

II. The Cusum Tests

Let the standard regression model be

$$y_t = x_t b + \epsilon_t,\ t = 1, ..., T, \tag{1}$$

where y_t is a (T x 1) vector of observations of the dependent variable, x_t is a (T x K) matrix of observations of K explanatory variables, b is a (K x 1) vector of parameters, and ϵ_t is a (T x 1) vector of disturbance term, which is assumed to follow a normal distribution with a zero mean and finite variance. The alternative model of structural change is

$$y_t = x_t b_t + \epsilon_t,\ t = 1, ..., T, \tag{2}$$

which permits the parameter b to change over time as denoted by the subscript t. The null hypothesis of parameter stability is then

$$H_0 : b_1 = ... = b_T = b. \tag{3}$$

When the data has a natural order like time, a simple way to examine stability of the parameters is to estimate the model recursively (Brown, Durbin, and Evans, 1975; Dufour, 1986). Using the first m observations in the sample, the initial estimate of b_m is obtained. Then, the sample size is gradually enlarged, adding one observation at a time to re-estimate b_r, r = m, ..., T, at each step. The sequence of estimated coefficients can be summarized as

$$b_r = (X_r'X_r)^{-1}X_r'Y_r,\ r = m, ..., T, \tag{4}$$

where $X_r' = [x_1, ..., x_r]$ and $Y_r' = [y_1, ..., y_r]$. This sequence of forward recursive estimates is more reliable near the end of the sample because of the increased sample size (and more degrees of freedom).

Recursive estimates of the coefficients are a descriptive device for assessing the influence of different observations in a sequential updating process. However, the elements of this sequence are strongly correlated even under the null hypothesis of stability. Therefore, it is necessary to derive an associated test statistics. A candidate is based on a sequence of k-step ahead prediction errors obtained as:

$$v_{rk} = y_r - x_r'b_{r-k}, \; r = m+k, ..., T, \quad 1 \leq k \leq T\text{-}1. \tag{5}$$

Since the v_{rk} have different variances, it is necessary to standardize them with respect to their own variances such that

$$w_{rk} = v_{rk}/d_{rk}, \; r = m+k, ..., T, \tag{6}$$

where, $d_{rk} = [1 + x_r'(X'_{r-k}X_{r-k})^{-1}x_r]^{1/2}$. The w_{rk} are called k-step ahead recursive residuals. Under the null hypothesis of no structural change, the residuals should be i.i.d. normal with mean zero and constant variance. Consequently, the null hypothesis of no structural change equivalent to (3) is

$$H_0: E(w_{rk}) = 0 \text{ and } E(w_{rk}^2) = \sigma^2, \text{ for all r.} \tag{7}$$

The intuition behind the hypothesis is as follows. Shocks on the fundamentals of the economy (e.g., taste, technology, demographic structure, and so on), and, therefore, changes in structural parameters lead to an increase in prediction errors. If the shocks are permanent, the change in parameters would be persistent, which in turn leads to systematically wrong predictions (in either positive or negative direction). On the other hand, if impacts of shocks are transitory and inconsistent, the prediction errors would change abruptly and have different variability over time. The Cusum test is sensitive to the size and direction of prediction errors, while the Cusum of squares test is designed to detect discontinuity in the size of prediction errors, i.e., different variability of prediction errors.

The Cusum test is literally based on the cumulative sums of k-step recursive residuals. The test statistic is defined as

$$W_{rk} = (1/\sigma)\Sigma^r_{j=m+1}w_{jk}, \; r = m+1, ..., T, \tag{8}$$

where $\sigma^2 = S_T/(T\text{-}m)$ and $S_T = \Sigma^T_r w_{rk}$. The null hypothesis of model stability is rejected at a significance level α if

$$\max |W'_{rk}| \; c_\alpha, \; r = m+1, ..., T, \tag{9}$$

where, $W'_{rk} = W_{rk}/\{(T\text{-}m) + 2[(r\text{-}m)/(T\text{-}m)^{1/2}]\}$. The critical values, c_α are 1.143 and 0.948 for the 1% and 5% levels of significance, respectively.

The statistic of the Cusum of squares test is based on the cumulative sums of squares series:

$$S_{rk} = (\Sigma^{r}_{j=m+1} w^2_{jk})/(\Sigma^{T}_{j=m+1} w^2_{jk}),\ r = m+1, ..., T. \qquad (10)$$

S_{rk} is a monotonically increasing positive number with $S_{Tk} = 1$. Under the null hypothesis of stability, $1\text{-}S_{rk}$ has a beta distribution and S_{rk} has the mean of $(r\text{-}m)/(T\text{-}m)$. Thus, the null hypothesis of no abrupt structural change is rejected if

$$|S_{rk} - (r\text{-}m)/(T\text{-}m)| > \delta_\alpha,\ \text{for any } r = m+1, ..., T. \qquad (11)$$

The critical values for δ_α are obtained from Table 1 of Durbin (1969) at $\alpha/2$ and $n = (1/2)(T\text{-}m)\text{-}1$ if $(T\text{-}m)$ is even, or by interpolating linearly between $n = (1/2)(T\text{-}m\text{-}1)\text{-}(1/2)$ and $n = (1/2)(T\text{-}m\text{-}1)$ if $(T\text{-}m)$ is odd.

III. Test of Structural Change with the GARCH Model

While the Cusum tests use rather complicated order statistics, the hypothesis of no structural change in equation (6) can easily be tested by the GARCH model, which estimates the mean and conditional variance simultaneously. For the test, the simple GARCH(1,1) model is enough, since the objective of the estimation is not to identify the correct lag structure, but to test whether the k-step recursive residuals have mean zero and constant variance [For detailed discussions about the GARCH process, see Engle (1982) and Bollerslev (1986)].

The GARCH(1,1) model for the k-step ahead recursive residual w_{rk} is specified as

$$\begin{aligned} &w_{rk} = \mu_k + e_{rk},\ r = m+1, ..., T, \\ &e_{rk}|\Phi_{r-1} \sim N(0, h^2_{rk}), \qquad (12) \\ &h^2_{rk} = \alpha_0 + \alpha_1 e^2_{(r-1)k} + \alpha_2 h^2_{(r-1)k},\ \ \alpha_0 > 0,\ \ \alpha_1 \geq 0,\ \ \alpha_2 \geq 0, \end{aligned}$$

where μ_k is the mean of w_{rk}, e_{rk} is the disturbance term which is allowed to have changing variance over time, Φ_{r-1} is past information available at time r, and h^2_{rk} is the variance conditional on past information. If α_1 and α_2 are zero, the disturbance term e_{rk} has a constant variance over time. Also, if the estimated mean, μ_k is zero, the null hypothesis of no structural change (smooth) in the model cannot be rejected. Consequently, the null hypothesis of no structural change can be rewritten in this context as

$$H_0 : \mu_k = 0 \text{ and } \alpha_1 = \alpha_2 = 0. \tag{13}$$

The test for the first part of the hypothesis is analogous to the Cusum test, while that for the second part is analogous to the Cusum of squares test.

IV. Data and Procedure

Annual time series data for the U.S. beef consumption during 1970-1987 are used for empirical analysis. For the beef consumption data, per capita civilian beef disappearance in pounds of retail weight from *Livestock and Meat Situation* are used. The retail prices of beef and pork, which are included as substitutes in this model, are obtained from the *Bureau of Labor Statistics*. For income, per capita personal disposable income (PDI) from the *Economic Indicator* are used. Prices and income are deflated by consumer price index (1967=100) for the homogeneity condition.

A semi-log model is employed in this study, which is appropriate for the study of food (Phlips, 1983). It should be noted, however, that it is hard to distinguish structural change from model misspecification (Alston and Chalfant, 1991). Thus, the Box-Cox function was estimated for model selection. The results supported the semi-log functional form.

Once parameters are estimated, they are used to calculate a sequence of k-period ahead forward prediction errors. Using this sequence, the Cusum and the Cusum of squares test statistics are derived. To calculate the test statistics, the parameters for k and m should be determined. This study uses k=1, i.e., one-step ahead prediction errors. Selecting m would be more complicated. A small value of m would provide a degrees of freedom problem for earlier estimates of b, while a large value of m would reduce the number of recursive residuals in constructing the distribution. This study uses m=5, which gives thirteen recursive residuals.

For the GARCH test, the sequence is employed to estimate the mean and the conditional variance simultaneously through maximum likelihood with the algorithm developed by Berndt, Hall, Hall and Hausman (1974).

V. Results and Interpretation

The empirical results show evidence of structural change in the model. Table 1 summarizes the recursive estimates of the model. The estimated coefficients of the price of beef ranges from -104.92 to -32.64, which indicates a significant change in the parameter over time. However, most changes took place at the first half of the examining period, i.e., 1975 to

TABLE 1 Recursive Estimates of the Semi-log Model

Year	Intercept	Beef Price	Pork Price	Income
1975	-366.94	-104.93	9.33	131.28
1976	-158.86	-66.85	8.78	70.99
1977	8.50	-49.08	16.79	23.24
1978	32.04	-49.64	18.36	16.51
1979	60.74	-54.04	22.47	8.45
1980	81.61	-55.61	27.05	2.78
1981	121.07	-51.31	32.49	-8.08
1982	161.42	-40.40	35.11	-19.40
1983	192.30	-36.17	36.19	-28.15
1984	185.48	-36.82	35.88	-26.23
1985	183.16	-37.21	35.83	-25.57
1986	195.12	-34.70	35.08	-29.05
1987	213.65	-32.65	33.99	-34.48
Mean	69.95	-49.96	26.72	6.33
S.D.	167.60	19.38	10.38	47.66

Note: Observations before 1975 were used for the initial estimates. Table 2. Estimated GARCH model

1982, after which the coefficients remain relatively stable. Similar results are shown for the price of pork. It ranges from 8.8 to 36.2 with most of change occurring between 1975 and 1982, as in the case of beef price. While the demand is less elastic with respect to its own price, it becomes more elastic with respect to the pork price.

The most significant structural change appears to have occurred with respect to income. The estimated coefficient ranges from 131.3 to -34.5, showing an apparent shift in income responsiveness. These results are consistent with Chavas' work (1983), which found a structural change during 1975 to 1979.

The estimated coefficients became negative after 1981. However, the Student-t statistics for those estimates mostly were not significantly larger than two. This study concludes the results conservatively, i.e., demand for beef in the U.S. is income inelastic rather than inferior.

The elasticities corresponding to these parameter estimates were in reasonable range for all cases. For example, the own price elasticities range from -1.23 in 1975 to -0.39 in 1987. The elasticities for pork price are 0.11 in 1975 and 0.41 in 1987, and those for income are 1.55 in 1975 and -0.41 in 1987.

While the estimated coefficients exhibit rough evidence of structural change in demand for beef, formal stability tests also support the evidence of structural change except for the Cusum of squares test. The calculated maximum Cusum statistic in absolute term is 0.97, which rejects the null of no systematic drift in the prediction error at the 5% level. However, the equivalent Cusum of squares statistic is -0.25, which does not reject no abrupt change in the prediction errors.

Table 2 reports the estimated results of the GARCH model. The

TABLE 2 Estimated GARCH Model

Parameter	Estimate	T-value
Mean Equation		
μ_k	- 2.610	- 50.48**
Variance Equation		
α_0	0.052	17.55**
α_1	0.998	2.59**
α_2	0.001	0.14

**(*) denotes significance at a 5(1)% level estimated mean of the recursive residual is -2.61 with the t-value of -50.48, which is significantly different from zero. The negative mean implies a consistent over-prediction. Even though the estimated α_2 is statistically insignificant, the estimate of α_1 is significantly different from zero even at the 1% level, which indicates that the variance is changing over time. The GARCH test rejects the null hypothesis of no structural change in the model during the sample period.

Interestingly, the sum of α_1 and α_2 is close to unity, which suggests that the variance drifts without no bound, i.e., integrated GARCH (Engle and Bollerslev, 1986; Nelson, 1990). This is analogous to the existence of a unit root in the first moment of distribution. The persistence in variance implies, in the present study, that the variance of the forecasting errors follows a random walk process and the abrupt changes in demand due to structural change cannot be predicted by past patterns.

VI. Summary and Conclusion

In demand analysis, one can examine the possibility of structural change as a test of the robustness of the model. A failure to capture structural change, when it exists, would lead to inadequate conclusions and inaccurate predictions of the economic behavior.

This study tested for structural change in the U.S. demand for beef during 1970 to 1987, in which significant economic and socio-demographic changes occurred. The Cusum and Cusum of squares tests were compared to the GARCH test newly applied in this study.

Results from the Cusum and the GARCH tests consistently rejected the null hypothesis of no structural change. However, the Cusum of squares test did not reject the null hypothesis of constant variability in the prediction errors. The Cusum and Cusum of squares tests appear to be somewhat sensitive to the model. Though not reported, no Cusum tests could reject the hypothesis of stability with a linear model. This evidence tends to support Garbade's finding of low power of test in the Cusum tests. The GARCH model appears to be a more powerful alternative of model stability test.

One limitation of this study is that the effects of other substitutes such as poultry or fish are not included because of degrees of freedom. To get more reliable results, those substitutes need be included in a future study.

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