EVALUATING POLICY ALTERNATIVES TO CONTROL NITRATE POLLUTION IN GROUNDWATER: A CONCEPTUAL APPROACH

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I. Introduction

Nitrate pollution of groundwater, created as a by-product of producing crops and livestock, is one of the most common agricultural pollution problems (De Haen, 1982). The nitrate groundwater pollution problem includes an externality aspect as well as an intertemporaneous aspect. Higher amounts of nitrogen fertilizer application yield a greater immediate profit to a farmer but also increase potential (negative) externalities on society by accumulating more pollution in the groundwater pool. Some of the nitrogen present in the soil in its nitrate form is leached, ultimately reaches groundwater and streams, and may finally damage[1] future users of the groundwater.

When externalities exist a competitive firm's behavior will not, in general, ensure an optimal allocation of resources, because the externality-producing firm imposes damages or costs to society that it does not consider in its profit maximizing decisions. A social optimum will result only if the externality costs are taken into account in the polluter's production decisions.

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1 A high nitrate concentration in groundwater may cause severe health problems through drinking water and food, which is reported as methemoglobinemia(blue-baby disease) in infants and gastric cancer in adults (Hanley, 1990)

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In the absence of regulations, however, producers would not have an incentive to limit nitrate pollution of groundwater and further to internalize the externalities caused from the pollutants. Imposing liability on producers for damages resulting from their actions would provide the proper incentives for them to reduce polluting activities. The use of liability as a control policy is most applicable to groundwater contamination from agriculture since it is an unilateral[2] externality problem (Segerson, 1990).

Groundwater quality can be considered as an exhaustible resource since the groundwater as a common property can be easily deteriorated by various human behavior, while it may be technically difficult or costly to improve the quality. Pollution control policy or regulations may be needed to encourage the producers to limit their polluting behavior to the socially acceptable level. Theoretical applications to the issue of pollution control on exhaustible resources have been found in Burness(1976), Levhari and Liviatan(1977), Heaps(1985), Dasgupta and Heal(1988), and Caputo(1990b). These papers have focused on the comparative analysis of the qualitative effect of various types of taxation on the optimal resource use. A common drawback of these papers is that their analyses are restricted to a model with one state variable.

Empirical applications of the groundwater pollution control have been found in Horner (1975), Anderson et al.(1985), Young and Crowder(1986), and Johnson et al.(1991). With the exception of Horner these papers do not take into account the externality aspect of the pollution or the internalization of the social cost caused by the pollutant. In addition, Young and Crowder, and Horner do not consider the dynamic(intertemporal) aspect of pollution.

This study is interested in developing a conceptual framework that can be readily applied in the real world decision environment of the groundwater pollution control. The objectives of this study are to develop a dynamic optimization model that incorporates the external and intertemporal aspects of nitrate pollution in groundwater and to

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[2] Groundwater contamination from nitrogen fertilizer is an example of unilateral "accident" because only the farmer can take steps to reduce the magnitude of contamination, i.e., the victim is unable to prevent contamination (Segerson, 1990)

evaluate the qualitative effects of alternative policy instruments on the groundwater quality and producer's profit. Section II develops a dynamic optimization model to control groundwater pollution from nitrogen. Section III establishes conditions of dynamic social optimality to find an optimal control policy on nitrate groundwater pollution, and then the analysis inquires whether the social optimality can be achieved by a decentralized market system. Optimal control theory is utilized for the analysis and the approach is partial equilibrium in nature. The following section evaluates the qualitative effects of alternative control policies on the optimal plan by applying comparative dynamic analysis of Caputo(1990a). Concluding remarks close the paper.

II. A Model to Control Nitrate Pollution in Groundwater

This section develops a dynamic optimization model in a partial equilibrium sense that incorporates the external and intertemporal aspects of nitrate pollution in groundwater. The model includes two control variables and two state variables. Applied water (irrigation) and nitrogen fertilizer are taken as the producer's control variables, considering that nitrate leaching is hastened by the action of water. Ambient concentration of nitrate pollution in the groundwater (or groundwater quality) and residual soil nitrogen are treated as state variables. The residual soil nitrogen is included as a state variable because its consideration by producers may influence the optimal plan of nitrogen application and the amount of nitrate leaching as well.

1. Model Formulation

Consider a farming sector with two variable inputs(nitrogen fertilizer, applied water), one output(corn), and n producers, where the producers share a common aquifer and are assumed to be identical and price-taking.

Crop yield function: Under given technology in this economy, producer i's (for any i, $1 \leq i \leq n$) crop yield at time t is determined by

$$Y(t) = Y(N(t), W(t), S(t)) \quad (1)$$

where, Y is the crop(corn) yield function, N is nitrogen fertilizer applied, W is applied water[3], and S is the residual soil nitrogen(N) in the root zone[4] that carried over from the previous time to the current time. Here nitrogen fertilizer N(t) and applied water(irrigation) W(t) are the producer's choice variables. It is assumed that the crop yield function Y(.) is concave and non-decreasing in (N,W,S). Then the price- taking producer's profit at time t is defined

$$\begin{aligned} \pi(t) &\equiv \pi(N(t), W(t), S(t)) \quad (2) \\ &= P^Y Y(N(t), W(t), S(t)) - P^N N(t) - P^W W(t) \end{aligned}$$

where, π is the profit function in the absence of regulation, P^Y is output price, P^N is nitrogen fertilizer price, and P^W is the price of water irrigated.

Pollution emission(or Nitrate leaching) function: While applying nitrogen fertilizer may increase crop yields and producer's profits, it also may result in nitrate leaching to the groundwater and negative externalities to society. Nitrate leaching function of the individual producer is defined in terms of available nitrogen in the soil(N+S) and applied water,

$$l(t) = l(N(t), W(t), S(t)) \quad (3)$$

where, l(t) is the amount of nitrate leaching during time t. It is assumed that the leaching function l(.) is convex and non-decreasing in (N,W,S), and $l_{NW} \geq 0$ $l_{SW} \geq 0$ with $l(0,w,0) = l(N,0,S) = 0$[5].

Ambient level of the pollution (or Groundwater quality):

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[3] Rainfall is assumed to be exogenously given in the model. Then applied water, which is the sum of irrigation water and rainfall, affects the crop yield function and nitrate leaching function. However, only irrigation water is treated as producer's control variable in the model.

[4] Residual soil nitrogen in the vadose zone is not considered to affect both the crop yield function and the nitrate leaching function, assuming that once nitrogen(or nitrate) penetrates beneath the root zone and presents in the vadose zone, it is treated as the previous leaching event and may reach the groundwater pool.

[5] The leaching function is assumed convex considering that as nitrogen or applied water increases, nitrate leaching may increase more rapidly because more of nitrogen or water remains in the soil profile. l(0,w,0) =l(N,0,S) = 0 implies that without either nitrogen(N+S) or water(W) in the soil no nitrate leaching occurs.

Ambient concentration of the pollutant (nitrate) in the common aquifer at time t is defined as the cumulative amount of nitrate leached[6] by the n producers from time 0 through t minus the cumulative amount of natural pollution decay in the aquifer,

$$A(t) = A(0) + \int_0^t [nl(N(\tau),W(\tau),S(\tau)) - \zeta A(\tau)]\, d\tau \quad \text{with} A(0) = 0 \tag{4}$$

[7]

where, A(t) is the ambient concentration of the pollutant in the aquifer at time t, and ζ is the natural nitrate pollution decay rate. It is assumed that the ambient nitrate concentration in the aquifer should never exceed a pre-specified baseline of the groundwater quality established by a regulatory authority[8], so that the groundwater can continue being used as a source of drinking water. Considering that the groundwater quality as an exhaustible resource mentioned earlier, the pre-specified baseline of the groundwater quality can be treated as the maximum allowed pollutability of the aquifer. That is, the allowed remaining pollutability in the aquifer is a negative indicator of the ambient pollution level. If more pollution stock is added to the aquifer then less pollutability remains for the future. Hence decision makers take into account the ambient level of the pollutant in the aquifer or the remaining pollutability in their decision. Then the allowed remaining pollutability stock of the aquifer at time t is defined as

$$\begin{aligned} R(t) &= R(0) - A(t) \\ &= R(0) - \int_0^t [nl(N(\tau),W(\tau),S(\tau)) - \zeta A(\tau)]\, d\tau \\ &\text{with } R(0) = \bar{R} \end{aligned} \tag{5}$$

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[6] Here we assume that once nitrogen is leached under the soil profile, it will eventually reach the aquifer and pollute the groundwater. However, a long time period may be required for the nitrate leached to reach the aquifer. The model does not incorporate this time lag.

[7] Equation (4) assumes that the volume of groundwater (or the reserve groundwater) in the aquifer is constant over time, which implies an approximate hydrologic equilibrium must exist between groundwater recharge and discharge in the system (U.S.G.S., 1974), and there is no nitrate inflow from other sources to the aquifer.

[8] It is implied that there exists a possible instrument to observe(monitor) the ambient pollution level at pre-specified points and the level will be announced to the public.

where, $\overline{R}$ is the pre-specified baseline of the groundwater quality in the aquifer at time zero or the (allowed) maximum pollutability of the aquifer, and R(t) is the allowed remaining pollutability of the aquifer at time t. Ignoring the natural nitrate decay process (denitrification) in the groundwater, i.e.$\zeta = 0$[9], for simplicity, the evolution of the remaining pollutability in the aquifer is determined by

$$\dot{R}(t) \equiv dR(t)/dt \\ = -nl(N(t), W(t), S(t)) \quad \text{with } R(0) = \overline{R} \tag{6}$$

This equation shows the pollution stock-flow relation. That is, the remaining pollutability stock decreases by the rate of nitrate leaching by all the producers.

Residual soil N: The residual N in the soil profile of the individual producer evolves by the following rule,

$$\dot{S}(t) \equiv dS(t)/dt \\ = N(t) - l(t) - U(t) \quad \text{with } S(0) = S_0 \tag{7}$$

where, U(t) is the amount of N plant-uptake during time t. Equation (7) implies N mass balance[10] in the soil profile. The residual soil N that carries over to the following time is the total available nitrogen at any time t (the sum of the residual soil N and the nitrogen fertilization at the current time t) less the amount used by plant-uptake and lost by nitrate leaching. Here the amount of N plant-uptake, U(t), is defined as a function of the crop yield Y (t), assuming that crop yield has a direct relationship with N plant-uptake.

$$U(t) \equiv U(Y(t)) \\ = U(N(t), W(t), S(t)) \tag{8}$$

Using equations (3) and (8), equation (7) is defined as a function of (N,W,S),

$$\dot{S}(t) \equiv g(N(t), W(t), S(t)) \\ = N(t) - l(N(t),W(t),S(t)) - U(N(t),W(t),S(t)) \tag{7a}$$

[10] Here organic N by mineralization, N loss by gas, and N addition from rainfall are ignored to avoid complexity.

Equations (6) and (7a) incorporates the dynamics of the system.

Nitrate Damage function: Negative externalities in the form of nitrate contaminated groundwater that is unfit for human consumption can be measured as social cost or damage to society.[11] The amount of damage from nitrates is defined as a function of the ambient level of the pollution[12],

$$D(t) = D(\, R(t)\,) \quad \text{with } D_R < 0,\ D_{RR} > 0 \tag{9}$$

where, D is the amount of social damage due to the pollutant, $D_R = \partial D(t)/\partial R(t)$ $D_{RR} = \partial^2 D(t)/\partial R(t)^2$. Note that D(.) is assumed to be convex in $\bar{R}$-R(t). Due to threshold effects of the pollutant, the damage function displays what scientists often call a non-linear dose response relation, where the amount of marginal damage from the unit pollutant is rapidly increasing in the neighborhood of the threshold point, say R=0. Then a socially optimal policy may involve keeping the pollution level from ever reaching R=0 (see Dasgupta, 1982).

Producers' liability: As a means of internalizing the social damage from nitrates, the producers(farmers) are liable for a certain share of the damage caused by their polluting behavior over time.

2. Socially Optimal Problem

In the absence of any regulations to control the pollution except the pre-specified baseline of the groundwater quality, the producers

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11 The measurement of the social cost or the damage may vary depending on the measurement method and site specific factors. A direct method of measuring the damage from nitrate is to evaluate the expected health costs that are incurred if the nitrate contaminated groundwater were used and the incidence of health problems occurs. An alternative method is to measure avoidance costs such as contaminated well treatment costs or costs of alternative water sources(e.g. bottled water). The avoidance cost method can be considered as the least cost method, whereas the direct method is the more expensive method(see Raucher, 1983).

12 Note that even when the groundwater contamination level is below the threshold point, the polluting behavior still imposes a negative externality to society by increasing the possibility of the future occurrence of the damage or by historically contributing for the future higher level of groundwater contamination. It is assumed, therefore, that the social damage is accrued from the nitrate polluted groundwater even below the threshold point.

would not bear the social damage or the negative externalities caused by the pollutant they discharge. Therefore individual producer's profit maximization would not yield social optimality. That is, socially optimal outcome is yielded when the polluters(producers) internalize the social cost or damage caused by the nitrate pollutant, which results from the n producers' behavior over time. We state the socially optimal problem as follows: Society(or the regulator) wants to maximize the n producers' profit stream over time reduced by the n producers' share of the social damage caused by the pollutant. This maximization assumes that the individual producers take into account the states of the (allowed) remaining pollutability stock in the common aquifer and the residual soil N of their farmland when they choose the optimal plan of N and W. Then the socially optimal problem is written as,

$$[P^s] \quad \text{Max} \int_0^T \{ n\,\pi(N(t),W(t),S(t)) - \alpha D(R(t)) \}\, e^{-\delta t}\, dt \qquad (10)$$

subject to

$$\dot{R}(t) = -nl(N(t),W(t),S(t)) \qquad (6)$$

$$\dot{S}(t) = N(t) - l(N(t),W(t),S(t)) - U(N(t),W(t),S(t)) \quad \text{for n producers} \qquad (7)$$

$$R(0) = \overline{R} \qquad (11a)$$

$$S(0) = S_0 \qquad (11b)$$

$$R(T) > 0 \qquad (11c)$$

where, α, $0 \le \alpha \le 1$, is the producers' liability share for the social damage from nitrates; δ is the continuous discount rate; T is a fixed planning horizon. Here $R(T) > 0$ implies that the groundwater quality in the aquifer should not be deteriorated more than the pre-specified baseline of the groundwater quality. Note that here the terminal time T is treated as a fixed and finite, thus is not subject to choice by the producer. It can be thought of as the length of a planning horizon that the regulator sets in order to achieve a certain goal during that time period. This model can be used to find an optimal control policy for nitrate pollution in groundwater.

III. Dynamic Social Optimality

1. Conditions for Social Optimality

To find the optimal conditions for social optimality, we introduce the Hamiltonian equation (see Kamien and Schwartz, 1981). The current value Hamiltonian for the socially optimal problem [P^s] is defined as (dropping out the time argument t)

$$H^s(N,W,R,S:\gamma^s,m^s) = n\,\pi(N,W,S) - \alpha D(R) - \gamma^s n l(N,W,S) + m^s n\{N - l(N,W,S) - U(N,W,S)\} \quad (12)$$

where, superscript "s" stands for the socially optimal problem, γ and m are the costate variables associated with the state variables R and S, respectively. γ may be interpreted as the decrease in the producer's profit caused by one more unit of nitrate pollutant in the aquifer, or the shadow cost of the pollution stock. Similarly, m may be interpreted as the increase in the producer's profit by carrying over one unit of residual nitrogen in the soil profile, or the shadow price of the residual soil N. Using the Maximum principle, the Hamiltonian conditions for a socially optimal outcome are (assuming an interior solution)

$$\partial H^s/\partial N: \pi_N(N,W,S) - \gamma^s l_N(N,W,S) + m^s\{1 - l_N(N,W,S) - U_N(N,W,S)\} = 0 \quad (13)$$

$$\partial H^s/\partial W: \pi_W(N,W,S) - \gamma^s l_W(N,W,S) - m^s\{l_W(N,W,S) + U_W(N,W,S)\} = 0 \quad (14)$$

$$\dot{\gamma}^s = \gamma^s\delta - \partial H^s/\partial R = \gamma^s\delta + \alpha D_R(R) \quad (15)$$

$$\dot{m}^s = m^s\delta - \partial H^s/\partial m = m^s\{\delta + l_S + U_S\} - \pi_S + \gamma^s l_S \quad (16)$$

$$\dot{R} = -\,n l(N,W,S) \quad (6)$$

$$\dot{S} = N - l(N,W,S) - U(N,W,S) \quad (7)$$

$$e^{-\delta T}\gamma^s(T) = 0,\ e^{-\delta T} m^s(T) = 0 \quad (17)$$

where, the subscripts denote the partial derivatives with respect to the variables. Each condition implies the following. Equation (13) states the optimal condition of nitrogen is such that, at each time t, nitrogen fertilizer is applied until the marginal contribution of nitrogen in producer's profit equates the net effects of unit of nitrogen on the marginal loss caused by nitrate leaching and the marginal gain resulted in carrying over the residual soil N to the following time. Similarly, equation(14) asserts that irrigation water is applied until the marginal contribution of water cancels off the marginal social cost of irrigation water. Condition (15) says that the shadow cost of nitrate increases faster than the discount rate, δ, by the marginal damage (negative externality) of nitrogen use to the society, while condition (16) shows that the shadow price of residual soil N increases by the discount rate plus the net effects of residual soil N on the producer's profit and nitrate leaching. Condition (17), which is called the transversality(end-point) condition, implies that any stock of the pollutability(R) or the residual soil N remaining at the terminal time T is worthless.

2. Controllability Of The Decentralized System : The Optimal Tax

Assume that we(or society) want to find an optimal control instrument that can accomplish social optimality with a decentralized system by imposing liability on each individual polluter(producer). Suppose there exists an optimal tax that ensures social optimality. Define the optimal tax scheme as a function of the ambient level of the pollution stock in the aquifer,

$$\Lambda(t) \equiv \Lambda(R(t)) \quad \text{with } \Lambda_R < 0 \tag{18}$$

where, Λ is the optimal tax rate and $\Lambda_R \equiv \partial\Lambda/\partial R$. Then an individual producer's problem under the optimal taxation is defined as

$$[P^i] \qquad \text{Max}_{\{N,W\}} \int_0^T \{\pi(N(t), W(t), S(t)) - \Lambda(R(t))\} e^{-\delta t} dt \equiv \int_0^T \{\pi(N(t), W(t), S(t)) - \alpha/n\, D(R(t))\} e^{-\delta t} dt \tag{19}$$

subject to
(6), (7), and (11a-c)

The current value Hamiltonian for the producer's problem under the optimal tax scheme is

$$H^x(N,W,R,S: \gamma^x,m^x) = \pi(N,W,S) - \Lambda(R) - \gamma^x l(N,W,S) + m^x \{ N - l(N,W,S) - U(N,W,S)\} \tag{20}$$

where, superscript "x" stands for the producer's problem under the optimal taxation. Assuming an interior solution and replacing the superscript "s" by "x" in equations (13)-(17), the Hamiltonian conditions for the problem [P^i] are the same as conditions (13)-(17) except condition (15) is changed as (21),

$$\dot{\gamma}^x = \gamma^x\delta - \partial H^x/\partial R = \gamma^x\delta + \Lambda_R(R) \tag{21}$$

It is then clear that conditions (21) and (15) have to be identical to ensure that the outcome of this problem is socially optimal. Therefore it can be said that social optimality can be achieved with the decentralized system only if condition (22) is satisfied,

$$\Lambda_R(t) = \alpha D_R(R(t)) \text{ for all } t \in [0, T] \tag{22}$$

From this result the following proposition can be stated.

> *Proposition:* The optimal tax schedule that ensures the social optimality has a marginal tax rate per unit of the pollutant that is equal to the producer's share of the marginal damage resulting from the unit of pollutant, for each time $t \in [0,T]$.

This is a so called Pigouvian tax in a dynamic sense. That is, the control of environmental pollution is best conducted with the help of a tax schedule that varies with the stock of the pollutants released by the producers. Such a tax scheme, however, might be hard to implement because it requires frequent monitoring of the nitrate level in the aquifer and frequent change of the tax rate corresponding to the pollution level. Administration of the socially optimal tax would likely raise many questions concerning the accuracy of the measurement of nitrate levels in the groundwater and whether the sources of the contamination were being identified. Given these difficulties, governmental units may want alternatives that are more easily administered, such as an input tax or an output tax. This raises the question of how well one of these instruments approximates the results of the socially optimal tax.

IV. Comparative Analysis of Policy Alternatives

The dynamic primal-dual approach developed by Caputo(1990a) can be applied to compare the qualitative effects of alternative policies on the groundwater quality and the producer's profit. The optimal tax, an output tax, and an input tax are considered as alternative policy choices, which are not under the producer's control. The perturbation of the producer's liability share implies a change of the optimal tax rate by (22). A decrease of the output price(P^Y) can be interpreted as an ad valorem tax on the output, while an increase of the N-input price(P^N) can be considered as an ad valorem tax on the input. Then define the vector of the policy variables under consideration as

$$\beta \equiv [\alpha; P^Y, P^N]$$

and let $J^*(\beta)$ be the optimal value function for the primal problem $[P^i]$, i.e.,

$$J^*(\beta) \equiv \int_0^T \{\pi(N^*(t:\beta), W^*(t:\beta), R^*(t:\beta), S^*(t:\beta):\beta) - \alpha/n\, D(R^*(t:\beta))\}\, e^{-\delta t}\, dt \quad (23)$$

where, (N,W,R,S) = ($N^*(t:\beta)$, $W^*(t:\beta)$, $R^*(t:\beta)$, $S^*(t:\beta)$) denotes an optimal solution to the problem $[P^i]$ when an arbitrary vector of β is chosen. Following Caputo's dynamic envelope theorem, we have

$$J_\alpha^*(\beta) \equiv \partial J^*(\beta)/\partial\alpha = -1/n \int_0^T D(R^*(t:\beta))\, e^{-\delta t}\, dt < 0 \quad (24a)$$

$$J_{Py}^*(\beta) \equiv \partial J^*(\beta)/\partial P^Y = \int_0^T e^{-\delta t}\, Y^*(N^*(t:\beta), W^*(t:\beta), S^*(t:\beta):\beta)\, dt > 0 \quad (24b)$$

$$J_{Pn}^*(\beta) \equiv \partial J^*(\beta)/\partial P^N = -\int_0^T e^{-\delta t}\, N^*(t:\beta)\, dt < 0 \quad (24c)$$

Result (24a) implies that an increase of the optimal tax rate via the producer's liability share α decreases the producer's optimal profit stream by paying more of the social damage over time. Result (24b) states that an output tax decreases the producer's profit stream by reducing the cumulative crop production, whereas (24c) says that the N-input tax decreases the producer's profit stream by reducing the factor (N-fertilizer) demand over time.

Considering that β appears linearly in the integrand of (19) and does not appear elsewhere in the system, we can apply Caputo's curvature theorem to the problem. Since $J^* \in C^{(2)}$ and is linear in β, the Hessian matrix of $J^*_{\beta\beta}(\beta)$ is symmetric positive semi-definite. Thus the diagonal elements of $J^*_{\beta\beta}(\beta)$ are non-negative and they are readily derived from (24):

$$J_{\alpha\alpha}^*(\beta) = -1/n \int_0^T D_R\ \partial R^*(t{:}\beta)/\partial\alpha\ e^{-\delta t}\,dt \geq 0 \tag{25a}$$

$$J^*_{Py,Py}(\beta) = \int_0^T e^{-\delta t} \{Y_N \partial N^*(t{:}\beta)/\partial P^Y + Y_W \partial W^*(t{:}\beta)/\partial P^Y + Y_S \partial S^*(t{:}\beta)/\partial P^Y \}\,dt \geq 0 \tag{25b}$$

$$J^*_{Pn,Pn}(\beta) = -\int_0^T e^{-\delta t}\ \partial N^*(t{:}\beta)/\partial P^N\,dt \geq 0 \tag{25c}$$

Recalling that $D_R < 0$, (25a) implies that $\int_0^T \partial R^*(t{:}\beta)/\partial\alpha\ e^{-\delta t}dt \geq 0$. That is when the producer's liability share α is increased under the optimal tax scheme, the ambient pollution level is decreased over the entire period. Result (25b) implies that the indirect effect of an output tax on the cumulative crop production is negative by changing the optimal path of N, W, and S, while (25c) says that the cumulative factor (N-fertilizer) demand is not increasing in its own tax.

Furthermore the symmetry of $J^*_{\beta\beta}(\beta)$ yields the following dynamic reciprocity condition:

$$\begin{aligned} J^*_{Py,Pn}(\beta) = \int_0^T e^{-\delta t} \{ &Y_N \partial N^*(t{:}\beta)/\partial P^N + \\ &Y_W \partial W^*(t{:}\beta)/\partial P^N + Y_S \partial S^*(t{:}\beta)/\partial P^N \}\,dt = \\ &-\int_0^T e^{-\delta t}\ \partial N^*(t{:}\beta)/\partial P^Y dt = J^*_{Pn,Py}(\beta) \end{aligned} \tag{26}$$

The result from (26) is not very informative unless we know the signs of $\partial W^*/\partial P^N$ and $\partial S^*/\partial P^N$. Suppose $\partial W^*/\partial P^N < 0$ and $\partial S^*/\partial P^N < 0$ (this implies W and S are complementary inputs to N, which are reasonable assumptions in this case), then (26) implies that

$$\int_0^T e^{-\delta t}\ \partial N^*(t{:}\beta)/\partial P^Y\,dt > 0$$

That is the output tax decreases the cumulative N-fertilizer demand, and thus results in a decrease of nitrate leaching over time.

The result from (24)-(26) can be summarized as follows: Not only an increase of the optimal tax rate via α but also an output tax or an input tax result in reducing the accumulation of the pollutant stock in the aquifer over the entire planning period, while all of these alternative tax policies decrease the producer's profit stream over the entire period.

V. Concluding Remarks

In this paper a dynamic optimization model of nitrogen management that incorporates the intertemporal and external aspects of nitrate groundwater pollution was developed by utilizing optimal control theory. A tax schedule that varies with the ambient pollution level and yields socially optimal outcome was obtained as an optimal control policy. Given difficulties of implementing this tax schedule, the qualitative effects of applying alternative policies such as an input tax or an output tax on the groundwater quality and the producer's profit were evaluated. It was qualitatively characterized that all of the pollution control policies including the optimal tax, an input tax , and an output tax had tradeoff effects on the producer's profit and the groundwater quality.

An important question is whether an input tax or an output tax as a second best control policy can produce an approximate outcome to social optimality. This quantitative question can only be answered by engaging in sensitivity analysis with an empirical data set. The sensitivity of the policy alternatives have been quantified by the authors by applying the model developed to a watershed in the vicinity of Westport, Minnesota in U.S.A. and will be reported in a separate paper. Applications to other locations remain for future research.

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