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THE DYNAMIC CONTROL POLICY FOR DISAPPEARING CROPS*

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I. Introduction

In agriculture, some crops suffer a serious non-stationarity problem which can be judged as dynamic instability. For example, the production of Korean barley, potato, sweet potato, millet, sorghum(kaoliang) have diminished dramatically for the last two decades. Some of them are already on the verge of extinction despite of the efforts to conserve them.[1] The purpose of this paper is twofold. The first is to build a model to explain the cause of diminishing production of disappearing crops. The second is to search policy alternatives which can stabilize the status of those crops in critical condition.

In macroeconomic stabilization theory, which was originated by Phillips (1954, 1957) and later combined with Tinbergen's macroeconomic policy theory (Tinbergen 1952), linear dynamic control methods have been used to explain and to solve dynamic instability problems. In the following we will introduce a brief outline of linear dynamic control theory.

For example, let us consider an economy which can be described by an autoregressive order one process of x_t such as

(1) $x_t = rx_{t-1} + u_t + \omega_t$

where r is a constant, u_t is a deterministic "forcing" variable and w_t is a white noise. A "forcing" variable is a exogenous variable which

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[1] The conservationist logic is well-known and not in the scope of this paper.

forces an autoregressive process to change. Let E_{t-k} be an expectations operator conditioned on all the information available up to period t-k. Define $E_{t-k}x_t$ as a short-period expectations for x_t when k is finite, and a long-period expectations for x_t when k is infinite. Suppose we are at period t-k and initial value x_{t-k} is given. From the solution of equation (1), we have

$$(2)\ x_t = r^k x_{t-k} + \sum_{j=0}^{k-1} r^j u_{t-j} + \sum_{j=0}^{k-1} r^j \omega_{t-j}.$$

For short-period expectations, set k=1 without loss of generality. Then the short-period expectations for x_t are

$$(3)\ E_{t-1}x_t = rx_{t-1} + u_t.$$

For the long-period expectations, set $k=\infty$. If $|r|<1$, the series will converge so that

$$(4)\ x_t = \sum_{j=0}^{k-1} r^j u_{t-j} + \sum_{j=0}^{k-1} r^j \omega_{t-j}.$$

Then the long-period expectations are

$$(5)\ E_{t-k}x_t = \sum_{j=0}^{\infty} r^j u_{t-j} \qquad k = \infty.$$

If $|r| < 1$ and $\omega_t=0$ for all t, the model can be called "static," since the long-period expectations are constant at $\sum_{j=0}^{\infty} r^j u_{t-j}$ and the short-period expectations are always fulfilled, i.e., $E_{t-1}x_t = x_t$. If $|r| < 1$ but $\omega_t>0$ for some t, the short-run expectations may not be fulfilled for some t. However, the condition $|r| < 1$ still holds so that the economy has a dynamic force which tends to push it back toward the constant long-period expectation point. We can call this model "stationary." If $|r|\geq 1$, the long-period expectations are not defined. In this case, the long-period expectations and the short-period expectations are inter-dependent since the effects of a shock, which can be described as a difference between the short-period expectations and the realization, do not die down as time goes by. We call this type of model "non-stationary." If $|r|\geq 1$, the economy is on a "razor's edge" so that any slight deviation from the dynamic equilibrium will bring about a permanent "shifting" away from the equilibrium. In this case a dynamic stabilization policy should concentrate on restraining the

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[2] All finite order autoregressive processes can be expressed by a first order vector autoregressive process.

absolute value of c from being greater than or equal to 1. If the time series of forcing variables (=u_t) are non-stationary but $|c| < 1$ so that x_t will converge to a weighted sum of non-stationary forcing variables ($=\sum_{j=0}^{\infty} r^j u_{t-j}$), a dynamic stabilization policy should concentrate on stabilizing the time series of the forcing variables.

In the second section of the paper, we will build a model which generates a long-run downward time path of production and will draw out a suitable dynamic stabilization policies by examining the causes of the downward trend.

II. The Model and the Policy Alternatives

The purpose of the section is to build a dynamic linear rational expectations model which accommodates the basic non-stationary features mentioned above so that it generates a long-run downward trend of production, and to observe the effects of stabilization policies on the time paths of the production of disappearing crops.

1. The Model

If there is a quantity adjustment cost, at period k-1, the maximization problem of a perfectly competitive producer who has an additively separable utility[3] and infinite time horizon is

$$(6)\ \underset{X}{\text{Max.}}\ E_{k-1}\sum_{j=0}^{\infty}\beta^j U(\pi_{k+j})$$

$$\text{s.t. } \pi_k = p_k(x_k+\varepsilon_{sk}) - C(x_k, d_k) - \frac{1}{2}\alpha(x_k-x_{k-1})^2$$

where β is the time discount rate such that $0 < \beta < 1$, α is a positive constant, $U(\cdot)$ is a utility function, $C(\cdot)$ is a cost function, $\frac{1}{2}\alpha(x_k-x_{k-1})^2$ is the adjustment cost, p_k is the relative price of the good, x_k is the planned quantity of the production at period k, ε_{sk} is a white noise which denotes the production risk, and the term d_k reflects technological progress such as

[3] We choose an additively separable utility function because it is immune to the time inconsistency problem in addition to its calculational convenience.

(7) $d_k = \tau d_{k-1} + \varepsilon_{\tau k}$

where $\varepsilon_{\tau k}$ is a white noise at period k and τ is a coefficient which denotes technical progress. The adjustment cost in the production could be justified when land preparation of a plot has crop-specific requirement(Eckstein, 1984). The cost function is quadratic such that

(8) $C(x_k, d_k) = \frac{1}{2} \gamma x_k^2 - \gamma d_k x_k$

where γ is a coefficient.

If we assume a linear utility function the producer's optimization problem would be

(9) $\underset{X}{\text{Max.}}\ E_{k-1} \sum_{j=0}^{\infty} \beta^j \{p_{k+j}x_{k+j} - C(x_{k+j}, d_{k+j}) - \frac{1}{2} \alpha(x_{k+j} - x_{k+j-1})^2\}$

Now the first order conditions are

(10) $E_{t-1}p_t - \gamma x_t + \gamma\tau d_{t-1} - \alpha(x_t - x_{t-1}) + \alpha\beta E_{t-1}(x_{t+1} - x_t) = 0,$
$t = k, k+1, k+2, \ldots$

Note that the technological progress has the same effects as the price increase. The transversality condition[4] is

(11) $\lim_{T \to \infty} \beta^T \{E_{T-1}p_t - \gamma x_T + \gamma\tau d_{T-1} - \alpha(x_T - x_{T-1})\} x_T = 0.$

Rewriting the first order condition equation, we have the producer's planned supply curve,

(12) $x_t = \frac{\alpha}{\alpha+\alpha\beta+\gamma} x_{t-1} + \frac{\alpha\beta}{\alpha+\alpha\beta+\gamma} E_{t-1}x_{t+1} + \frac{1}{\alpha+\alpha\beta+\gamma} E_{t-1}\, p_t + \frac{\gamma\tau}{\alpha+\alpha\beta+\gamma} d_{t-1}$

If all the producers are identical we can think the crop is produced by one big producer. Then the market supply curve of the crop is the same as the producer's supply curve (12). If there is a government's intervention in the supply side, g_t, the market supply curve is

(13) $x_t^s = x_t^s + g_t + \varepsilon_{st}$

where ε_{st} is a white noise at t.

On the other hand, if we assume the market demand curve is

(14) $x_t^d = b_0 - b_1 I_t - b_2 p_t + \varepsilon_{dt}$

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[4] In order for the transversality condition to be met we need $\tau < \frac{1}{\sqrt{\beta}}$ and $\upsilon < \frac{1}{\sqrt{\beta}}$. The coefficient υ will be explained in equation (15).

where b_0, b_1, b_2 are positive constants. I_t is the consumers' real income such that

(15) $I_t = \upsilon I_{t-1} + \varepsilon_{It}$.

Here, p_t is the relative price, ε_{It} is a white noise and υ is a coefficient which denotes income growth. Note that we make the crop an inferior good by assuming b_1 a positive constant.

If we assume the producers and the consumers have rational expectations and the ex ante equilibrium condition $E_{t-1}x_t^d = E_{t-1}x_t^s$, we can calculate $E_{t-1}p_t$ from (12), (13) and (14). That is,

$$(16)\ E_{t-1}p_t = \frac{\alpha+\alpha\beta+\gamma}{1+(\alpha+\alpha\beta+\gamma)b_2}\{b_0 - b_1 I_t - \frac{\gamma\tau}{\alpha+\alpha\beta+\gamma}d_{t-1} - \frac{\alpha}{\alpha+\alpha\beta+\gamma}x_{t-1} - \frac{\alpha\beta}{\alpha+\alpha\beta+\gamma}E_{t-1}x_{t+1} - g_t\}$$

where E_{t-1} is an expectation operator conditioned on the information set Ω_{t-1} such as

$$(17)\ \Omega_{t-1} = \{x_t, x_{t-1},\ldots; I_t, I_{t-1},\ldots; p_{t-1}, p_{t-2},\ldots; d_t, d_{t-1},\ldots; \alpha, \beta, \gamma, \upsilon, \tau, b_0, b_1, b_2\}.$$

At period t-1, the farmer decides the production quantity of the next period($=x_t$), has the information about the production technology of the next period which includes d_t, and knows I_t which is determined at the end of period t-1.

Plugging (16) into (12), we have

$$(18)\ E_{t-1}x_{t+1} - \frac{1+(\alpha+\alpha\beta+\gamma)b_2}{\alpha\beta b_2}x_t + \frac{1}{\beta}x_{t-1} = \frac{b_1}{\alpha\beta b_2}I_t - \frac{\gamma\tau}{\alpha\beta}d_{t-1} + \frac{1}{\alpha\beta b_2}g_t - \frac{b_0}{\alpha\beta b_2}.$$

Let g_t be a feedback policy such as

(19) $g_t = c_0 + c_1x_t + c_2x_{t-1} + c_3I_t + c_4d_{t-1}$.

Then the difference equation which represents the time path of the producer's optimal production plan is

$$(20)\ E_{t-1}x_{t+1} - f_1x_t + f_2x_{t-1} = \frac{b_1+c_3}{\alpha\beta b_2}I_t - \frac{\gamma\tau b_2 - c_4}{\alpha\beta b_2}d_{t-1} - \frac{b_0-c_0}{\alpha\beta b_2}.$$

where

$$(21)\ f_1 = \frac{1+(\alpha+\alpha\beta+\gamma)b_2+c_1}{\alpha\beta b_2},$$

$$f_2 = \frac{\alpha b_2 - c_2}{\alpha\beta b_2}.$$

Now, assuming the producer determines optimal value of x_t using the information in Ω_{t-1}, let us solve the difference equation (20) for x_t as follows. Following Sargent (1987, p.395), let us define a lag operator L_Ω such as

$$(22)\ L_\Omega^{-i} E_{t-1} x_t = E_{t-1} x_{t+i}.$$

The lag operator makes us to use the same information set as Ω_{t-1} in expecting $\{x_{t+1}, x_{t+2}, x_{t+3}, \dots\}$. Then we can rewrite equation (20) such as

$$(23)\ (L_\Omega^{-2} - f_1 L_\Omega^{-1} + f_2) E_{t-1} x_{t-1} = \frac{b_1 + c_3}{\alpha\beta b_2} I_t - \frac{\gamma\tau b_2 - c_4}{\alpha\beta b_2} d_{t-1} - \frac{b_0 - c_0}{\alpha\beta b_2}$$

Factoring the equation, we have

$$(24)\ (L_\Omega^{-1} - \delta_1)(L_\Omega^{-1} - \delta_2) E_{t-1} x_{t-1} = n_{t-1}$$

where $(\delta_1 + \delta_2) = f_1$, $\delta_1\delta_2 = f_2$, and

$$(25)\ n_{t-1} = \frac{b_1 + c_3}{\alpha\beta b_2} I_t - \frac{\gamma\tau b_2 - c_4}{\alpha\beta b_2} d_{t-1} - \frac{b_0 - c_0}{\alpha\beta b_2}.$$

Since $E_{t-1} x_t = x_t$ and $E_{t-1} x_{t-1} = x_{t-1}$,

$$(26)\ x_t - \delta_1 x_{t-1} = -\frac{1}{\delta_2 \left(1 - \frac{L_\Omega^{-1}}{\delta_2}\right)} n_{t-1}.$$

If $|\delta_2| > 1$,

$$(27)\ x_t - \delta_1 x_{t-1} = -\frac{1}{\delta_2}\left\{ \frac{b_1 + c_3}{\alpha\beta b_2} \sum_{i=0}^{\infty} \left(\frac{\upsilon}{\delta_2}\right)^i I_t - \frac{\gamma\tau b_2 - c_4}{\alpha\beta b_2} \sum_{i=0}^{\infty} \left(\frac{\tau}{\delta_2}\right)^i d_{t-1} \right\} + \frac{1}{\delta_2 - 1} \frac{b_0 - c_0}{\alpha\beta b_2}$$

If $|\upsilon| < |\delta_2|$ and $|\tau| < |\delta_2|$,

$$(28)\ x_t - \delta_1 x_{t-1} = -\frac{1}{\delta_2 - \upsilon} \frac{b_1 + c_3}{\alpha\beta b_2} I_t + \frac{1}{\delta_2 - \tau} \frac{\gamma\tau b_2 - c_4}{\alpha\beta b_2} d_{t-1} + \frac{1}{\delta_2 - 1} \frac{b_0 - c_0}{\alpha\beta b_2}$$

If $|\delta_1| < 1$, $|\delta_1| < |\upsilon|$ and $|\delta_1| < |\tau|$, we can solve the first order difference equation (28) such as

$$(29)\ x_t = \frac{\upsilon}{(\upsilon - \delta_1)(\upsilon - \delta_2)} \frac{b_1 + c_3}{\alpha\beta b_2} \{I_t - \sum_{i=0}^{\infty} \delta_1^i \varepsilon_{It-i}\} - \frac{\tau}{(\tau - \delta_1)(\tau - \delta_2)} \frac{\gamma\tau b_2 - c_4}{\alpha\beta b_2} \{d_{t-1} - \sum_{i=0}^{\infty} \delta_1^i \varepsilon_{\tau t-i}\}$$

$$- \frac{1}{(\delta_1-1)(\delta_2-1)} \frac{b_0 - c_0}{\alpha\beta b_2}$$

So far we have obtained the optimal solution for x_t by expanding difference equation (24) backward with the root whose absolute value is less than one and forward with the root whose absolute value is greater than one. It is because the producer's decision process for optimum x_t is a mixture of backward looking behavior and forward looking behavior.

Equation (29) shows that x_t is non-stationary when at least one of the forcing variables, I_t and d_{t-1}, is non-stationary unless I_t and d_t are cointegrated. Especially, if there is no government policy(i.e., $c_0=c_1=c_2=c_3=c_4=0$), and b_1 is positive(i.e., x_t is an inferior good), and is very small(i.e., there is no significant technological progress), x_t will go down to zero. In other words, the crop will disappear.

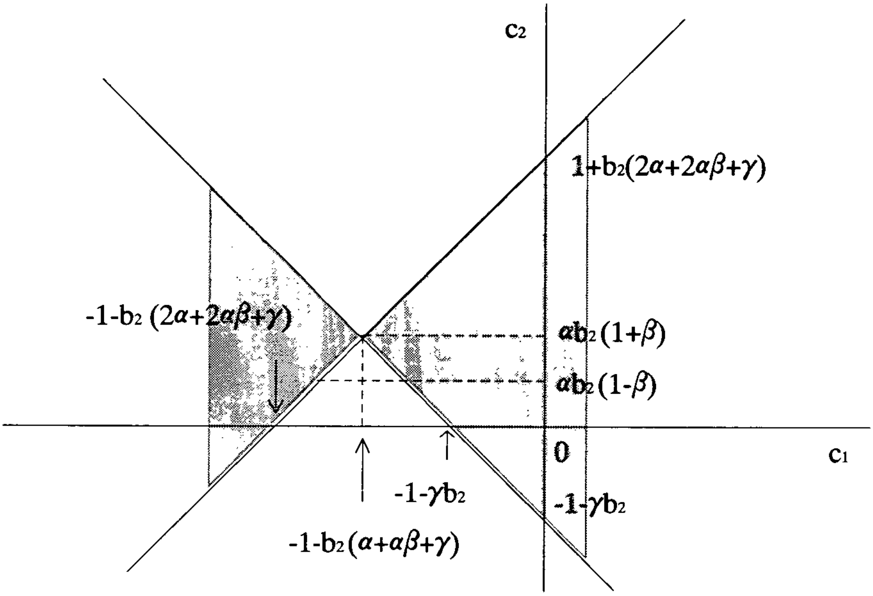
2. The Policy Alternatives

Now, using the results calculated above, let us develop policy alternatives through which the government could stabilize the downward trend of the production. A set of conditions which guarantees a stable solution for the time series x_t can be summarized as:

1) $|\delta_1| < 1$, $|\delta_2| > 1$ and
2) $|\upsilon|$, $|\tau| < |\delta_2|$ and
3) n_{t-1} is a stationary time series. In other words, at least one of the conditions from a) to e) is satisfied.
   a) the forcing variables I_t and d_{t-1} are both stationary (i.e., $|\upsilon|$, $|\tau| < 1$).
   b) $\frac{b_1+c_3}{\alpha\beta b_2} = 0$ when I_t is non-stationary but d_{t-1} is stationary (i.e., $|\upsilon| \geq 1$, $|\tau| < 1$).
   c) $\frac{\gamma\tau b_2 - c_4}{\alpha\beta b_2} = 0$ when d_{t-1} is non-stationary but I_t is stationary (i.e., $|\upsilon| < 1$, $|\tau| \geq 1$).
   d) I_t and d_{t-1} are both non-stationary (i.e., $|\upsilon|$, $|\tau| \geq 1$) and $\frac{b_1+c_3}{\alpha\beta b_2} = \frac{\gamma\tau b_2 - c_4}{\alpha\beta b_2} = 0$.

e) I_t and d_{t-1} are both non-stationary (i.e., $|u|, |t| \geq 1$) but cointegrated so that $\frac{b_1+c_3}{\alpha\beta b_2} I_t - \frac{\gamma\tau b_2 - c_4}{\alpha\beta b_2} d_{t-1} = s_t$
where s_t is a stationary series.

FIGURE 1



Now, let us assume that the government can control c_1, c_2, c_3, c_4, and α. The shaded region in Figure 1 shows the values of c_1 and c_2 which satisfy inequalities $|\delta_1| < 1$ and $|\delta_2| > 1$ when the assumptions α, γ, $b_2 > 0$ and $1 > \beta > 0$ are satisfied. Note that the point which satisfies the condition $c_1=0$ and $c_2=0$ belongs to the shaded region, i.e., even though we do not take any policy action, condition 1) is satisfied automatically.

Since we know the stability condition 1) is satisfied naturally, the focus of the stabilization policy would be on the conditions 2) and 3). It is most probable that at least one of I_t and d_{t-1} is not stationary because I_t denotes real income and d_{t-1} denotes technological progress.

If one of I_t and d_{t-1} is non-stationary, the government should make the coefficient of the non-stationary series zero. If I_t is non-stationary, the government could make

$$(30)\ \frac{b_1+c_3}{\alpha\beta b_2} = 0$$

by setting $c_3 = -b_1$ or $\alpha = \infty$. If d_{t-1} is non-stationary, the government could make

$$(31)\ \frac{\gamma\tau b_2 - c_4}{\alpha\beta b_2} = 0$$

by setting $c_4 = \gamma\tau b_2$ or $\alpha = \infty$.

c_3 and c_4 are the coefficients of I_t and d_{t-1} in the feedback policy equation (19), respectively. Hence the government could absorb the effects of the non-stationary forcing variable I_t and d_{t-1} on x_t through adjusting c_3 and c_4. In equation (6), α determines the cost of changing output quantity. Therefore $\alpha = \infty$ means the government prohibits transfer of resources from the production of disappearing crops to other uses.

If both I_t and d_{t-1} are non-stationary but happen to be integrated of the same order, there exists a coefficient ξ such as

$$(32)\ s_t = I_t - \xi d_{t-1}$$

where s_t is a stationary time series. Then from equation (25), we know that the government could make n_{t-1} stationary by adjusting c_3 and c_4 such that

$$(33)\ \frac{\gamma\tau b_2 - c_4}{b_1+c_3} = \xi.$$

In other words, when I_t and d_{t-1} are cointegrated, the government should adjust c_3 and c_4 at the same time to stabilize the downward trend of x_t. For example, if the government sets c_3 and c_4 to satisfy the following equation which is derived from (33), the government can stabilize the downward trend of x_t.

$$(34)\ c_4 + \xi c_3 = \gamma\tau b_2 - \xi b_1$$

III. Summary and Conclusion

The important results of the paper are as follows. First, we found that the downward trend of disappearing crop production is caused by non-stationarity of the forcing variables. For example, if the forcing variable I_t(income) is non-stationary and the crop is an inferior good, the income growth can make the crop disappear. Second, the government can neutralize the effects of the non-stationary forcing variables through a feedback policy which counterbalance the growth of the forcing variables. For example, the government can freeze the production quantity of x_t by setting the level of coefficients of I_t and d_{t-1} in equation (29) at zero. Third, if the forcing variables are cointegrated, the government can use a feedback policy which makes the effects of the forcing variables offset each other. In other words, if the government can compensate 'inferior' crop growers' income through investment to the technological development, the government can prevent the inferior crop from disappearing. Fourth, yet another stabilization method for the government is to enforce prohibitively high production quantity adjustment cost so that the producers cannot change the production quantity. For example, taking advantage of "The Convention on Biological Diversity," the government can legislate a conservation law which restrains transfer of land from production of disappearing crops to other uses and implements subsidy policies to encourage production of disappearing crops.

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