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THE STABLE DISTRIBUTION OF AGRICULTURAL CASH PRICE CHANGES

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ABSTRACT

The Gaussian assumption is easy to apply to economic analyses and many of its properties have been revealed to economists so that the distributional hypothesis has been embraced in economic and financial analyses, despite the fact that empirical evidences show distinct anomalies from the normal distribution. This study shows that the distributions of major U.S. agricultural commodity cash price changes are significantly different from normality. They have fatter tails and hither peaks than the normal distribution. The leptokurtic behaviors of the cash price changes are effectively captured by the stable distribution rather than the normal distribution. At the same time, when real data correspond to the stable distribution, variance may not be an effective scale parameter of the data. This raises a necessity of finding other alternatives to the traditional second moment.

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I. Introduction

In the analyses of futures hedging or option pricing models, it has been assumed that price changes follow the normal distribution. If the maintained assumption are not true for the data at hand, albeit the constructed model is well supported by the theory and other empirical estimation procedures are solid, there might be misleading results. This suggests that, in empirical studies, discovering the best fitted distributional model for the data is as important as constructing a sound model and finding a relevant data set.

Commodity and financial time series have been traditionally assumed to follow the normal distribution with finite mean and variance. The Gaussian assumption is easy to apply to economic analyses, and many of its properties have been revealed to economists so that the distributional hypothesis has been embraced in economic and financial analyses, despite the fact that empirical evidences show distinct anomalies from the normal distribution. Empirical studies have found that financial data have distributions with fatter tails and higher peaks than the normal distribution, known as leptokurticity. A class of studies has tried to discover a suitable alternative to the normal distribution to explain the abnormality, and has found that the stable distribution is a suitable alternative to explain the fat-tails behavior of financial data (c.f., Fama 1965a; Fama and Roll 1971; Fielitz and Rozelle 1983; Hall, Brorsen and Irwin 1989; Gribbin Harris and Lau 1992; McCulloch 1996 Nolan and Panorska 1997; and Nolan 1999).

The stable distribution is a more generalized distributional model in which the normal distribution is a special case. With different values of the parameters, the stable distribution corresponds to different distributions with dissimilar characters. The stable distribution is a generalization of the Brownian motion.[1] It was

[1] The word stable is used because the shape of the distribution is stable or unchanged under the sums of same type processes. That is, if two

developed by Paul Levy in the 1920s by relaxing the finite variance condition of the Brownian motion. It has infinite (undefined) variance, and it is discontinuous except for the special case of the Brownian motion. It was studied extensively by Fama (1963; 1965a; 1965b) and Mandelbrot (1963a; 1963b) in the 1960s. They suggest that the distributions of stock returns are significantly different from normality, and the non-normality behavior is well captured by the stable distribution.

Helms and Martell (1985) examined the distributions of price and log-price changes in various futures contracts, concluding that the distributions are not normal and amore generalized and sophisticated approach is, therefore, required to determine the exact nature of the data-generating process. Using the stability-under-addition test,[2] Cornew, Town, and Crowson (1984) and So (1987) confirm that a better correspondence for futures price changes is usually obtained when using the stable distributions; i.e., the stable distribution adequately well describes futures price changes compared to the performance of the normal distribution. Contrary to those results, Hall, Brorsen, and Irwin (1989) and Gribbin, Harris, and Lau (1992) report that the stable distributions are not quite suitable for futures price changes and they suggest a mixture of normal distributions[3] as a good alternative. The futures

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independent random variables with the same type of the distribution are combined linearly, then the resulting distribution is also a random variable with the same type. There are several different names for the stable distribution: Pareto-Levy distribution, stable Paretian distribution, stable family distribution, and stable distribution.

2 Using the fact that a stable distribution is invariant under addition, Fama (1963) suggest that the distribution of sums of the stable distribution is also a stable distribution with the same values of α, which is the most important parameter in the stable distribution and is called the characteristic exponent. Thus, the estimated values of α must be equal across the sums, provided that the underlying leptokurtic distribution is stable. Whether or not the stable distribution effectively explains the leptokurticity has been extensively tested using the stability-under-addition test (c.f., Fama and Roll 1971; Fielitz and Rozelle 1983).

3 The model suggests that the movements of stock returns can be

price data used in the divergent studies are different, suggesting that different data may produce different results. However, neither of the two different results suggests that the normal distribution does a good job in capturing the behavior of financial time series.

The objective of this study is to test whether or not the cash price changes in agricultural commodities are more effectively explained by the stable distribution than the normal distribution when they exhibit fat-tails behaviors. According to studies of commodity prices, such as Williams and Wright (1991) and Chambers and Bailey (1996), the time series of commodity cash prices have two common features. First, they display considerable positive autocorrelation; periods with high prices tend to follow periods with high prices, and low prices tend to follow low prices. Second, they have spikes-periods when the price jumps abruptly to a very high or low level relative to its long-run average. The second property corresponds to the fat-tails behavior of financial data. This suggests that price changes are not distributed normally, and fat-tails and peakedness characterize many price change distributions. This paper will be the first empirical offering for the agricultural price changes, confirming the non-normality and leptokurticity and further showing that the stable distribution is a better distributional model for such price changes. At the same time, it discusses the usefulness of variance as a measure of dispersion when the price changes correspond to the stable distribution. This study deviates from traditional agricultural economics studies by looking for an alternative to the normal distribution and the second moment, variance.

This study provides empirical contributions in at least three areas. First, finding that leptokurticity is explained poorly by the traditional normal distribution and instead explained well by the stable distribution might provide new insights in the economic analyses of agricultural financial and commodity markets. Second, under the stable distribution, variance may be undefinable and it

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represented by combinations of normal distributions, possessing different variances and possibly different means.

may not effectively describe the dispersion of the data in analysis. This raises the necessity of finding other alternatives to the traditional second moment, variance. Third, the comparison of efficiency between the two competing distributional models (the normal and stable distribution) in capturing the behaviors of financial and economic time series has been elusive in the previous studies which use the stable distribution for financial data analyses. This study performs the comparison by estimating the fitted stable density and fitted normal density and by comparing them with the smoothed data density. A recently brewed method by Nolan (1997) is used to calculate the stable density for the price changes.

The remainder of the study proceeds as follows. The stable distribution and estimation procedure are explained in the second section. Data used in this study are explained in the third section and empirical results are subsequently presented in the fourth section. The study ends with concluding remarks in the last section.

II. A Brief Review of the Stable Distribution

The stable distribution has four parameters which can be estimated through the characteristic function of the stable distribution.[4] The estimates of the parameters help to confirm whether the distribution of a random variable corresponds to a normal distribution or to a broader set of the stable distributions. They also tell us about the behavior of the series: peakedness, skewness, location, and scale of the distribution. There are three special cases where one can write down closed form expressions for the density of the stable

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[4] The characteristic function is the Fourier transform of a random variable, y,i.e., E[exp(i-y-t)] for t, where t is any real number, i denotes the square root of -1, E[·] is a mathematical expectation operator, and exp (·) denotes the exponential function. The Fourier transform is used to define the stable distributions, since with certain exceptions the probability distribution of the stable processes cannot be specified explicitly.

distribution and verify directly that they are stable: normal, Cauchy, and Levy distributions.[5] Other than these three distributions, there are no known closed form expressions for the stable densities. Instead, the stable distribution can be characterized by its characteristic functional form as follows:

(1) $F(t) = E[\exp^{iyt}] = \exp[i\delta t \ -\gamma|t|^{\alpha} \ (1+i\beta(t/|t|) \ w(t, \ \alpha))]$,

where y is a random variable; i is $\sqrt{-1}$; t is any real number; and $w(t, \alpha)$ is $(2/\pi)$ $(\tan|t|)$ if $\alpha=1$, otherwise $w(t, \alpha)$ is $\tan(\alpha\pi/2)$.

The function has four parameters; α, β, δ and γ. The first parameter, $\alpha\in(0, 2]$, is the characteristic exponent that accounts for the relative importance of the tails. It measures the peakedness of the distribution as well as the fatness of the tails. If α is equal to 2, then the distribution corresponds to the normal distribution with finite mean and variance. When α is in (1, 2], the random variable has finite mean. The second parameter, $\beta\in[-1, 1]$, is the skewness parameter. In particular, when β is equal to 0, the distribution is symmetric. When β corresponds to+1 (-1), the distribution is fat-tailed to the right (left), or skewed to the right (left). The degree of right skewness increases as β approaches +1, and vice versa. As α approaches 2, loses its effect and the distribution approaches the symmetric normal distribution regardless of the value of β. The third parameter, $\gamma\in(0, +\infty)$, is the scale parameter. It compresses or extends the distribution from the point of its location parameter. The last parameter, $\delta\in(-\infty, +\infty)$, is the location parameter. It shifts the distribution to the left or right.

If tails of a distribution are heavier than those of a distribution which has exponentially decreasing tails, it is said that

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[5] Setting $\alpha=2$, $\beta=0$, $\delta=\mu$, and $\gamma=\sigma^2/2$ in Equation 1 yields the characteristic function of the normal distribution. Cauchy distribution is the case with $\alpha=1$, $\beta=0$, $\gamma\in(0, +\infty)$, and $\delta\in(-\infty, +\infty)$. Levy distribution is the case with $\alpha=1/2$, $\beta=1$, $\gamma\in(0, +\infty)$, and $\delta\in(-\infty, +(\infty)$.

the distribution has heavy tails, or fat tails. A statistical result of the fat-tails is that not all moments may exist. When a distribution has sufficiently long tails, the first few moments will not characterize the distribution because they diverge. Under the fat-tails, mean and variance may be undefined or infinite and therefore unsuited as effective descriptions of the distribution. Cornew, Town, and Crowson (1984) provide a mathematical proof for the possibility of the undefined moments for a distribution with fat-tails. Further, they show through a trading performance analysis under both the normal and stable distribution that when the traditional first and second moments, mean and variance, are not definable under a specific stable distribution, the location and scale parameters of the stable distribution, γ and δ, can be suitable alternatives.

To estimate the stable estimation for a time series, one needs some initial values of the four parameters. This study adopts McCulloch's (1986) quantile estimation methodology to estimate the initial values. With the quantile estimators as initial approximations to the parameters, a constrained quasi-Newton method is used to maximize the log-likelihood function for an i.i.d. stable sample of a random variable, X_1, X_2, $\cdots$, X_n, as follows:

(2) $l(\omega)=\sum_i \log f(X_i|\omega)$,

where $f(X_i|\omega)$ is the density function of thestable distribution, denotes the parameter vector by $\omega\equiv(\alpha, \beta, \gamma, \delta)$ in a parameter space $\Omega=(0, 2]\times[-1, 1]\times(0, +\infty)\times(-\infty, +\infty)$, and the quasi-Newton method is constrained by the parameter space.

To estimate the parameters by maximizing the log-likelihood function, $f(X_i|\omega)$ must be computed for each observation. There have been several efforts to compute the stable density, such as Holt and Crow (1973), Panton (1992), and Nolan (1997). Among them, Nolan provides an efficient numerical computation method for the stable density and distribution function. This study adopts Nolan's methodology.

DuMouchel (1973) showed that when the parameter vector, ω, is on the interior of the parameter space, Ω, the maximum likelihood estimator follows the standard theory so that it is consistent and asymptotically normal. If ω is near the boundary of Ω, the finite sample behavior of the estimator is not precisely known, because the distribution of the estimator may be skewed away from the boundary. Further, in this case, the asymptotic normal distribution ofthe estimator tends to be a degenerate distribution at the boundary point.

The stable distribution has been used in literature, but testing whether the distribution fits real data better than the normal distribution has been elusive. However, such a test is necessary to increase credibility of empirical results. One method to determine whether the data are consistent with the stable distribution is to plot a smoothed density of the data and compare it with the fitted stable density alongside the fitted normal density, as proposed by Nolan (1999). Clear multiple modes or gaps in the smoothed density are evidence that the data do not come from a stable distribution. The Gaussian kernel can be used to obtain the smoothed density of the data (c.f., Pagan and Hong 1990 and Campbell, Lo and MacKinlay 1997). The density plots indicate whether the fitted stable density matches the real data better than the fitted normal density near the mode and tails of the distribution. If the leptokurtosis and skewness are better described by the stable density than by the normal density, then one can say that the data set follows a non-normal distribution and is better fitted to the stable distribution.

III. Data

The time series data used in this study are daily cash prices for 14 different agricultural commodities. The data include broilers, cocoa, coffee, corn, large white eggs, Kansas City wheat, oats, soybeans, soybean meal, soybean oil, spring wheat, sugar #11, wheat, and wheat #1 cash prices. The market location for cocoa, coffee, and sugar #11 is the New York Board of Trade (NYBOT);

the market for Kansas City wheat is the Kansas City Board of Trade (KCBOT); and market for all other commodities is the Chicago Board of Trade (CBOT). The series are daily, and most begin on August 28, 1992 and end on March 8, 2000. Soybean oil starts on June 2, 1969; spring wheat starts on February 1, 1983; and broilers and large white eggs begin on December 1, 1991. The number of observations is 7,766 for soybean oil, 4,325 for spring wheat, 2,068 for broilers and large white eggs, and 1,888 for other commodities.

Summary statistics of the cash price changes are presented in Table 1. Most series have negative skewness, indicating that the distributions of price changes are skewed to the left. All the commodities have kurtosis that exceeds 3.0, indicating that the distributions have higher peaks and fatter tails than the normal

TABLE 1. Descriptive Statistics for Agricultural Commodity Cash Price Changes.

Variables	Mean	Std. Dev.	Skewness	Kurtosis	Normality Test
Broilers	-7.2667	1.0117	-0.34	13.21	9028.19 (0.00)
Cocoa	-0.1420	23.0537	0.43	5.71	641.18 (0.00)
Coffee	0.0401	4.6437	1.46	38.21	97295.14 (0.00)
Corn	-0.0064	4.0924	-0.90	12.25	6928.95 (0.00)
Large White Eggs	-0.0065	1.4306	-0.37	8.92	3070.04 (0.00)
Kansas City Wheat	-0.0274	6.1361	-0.11	7.27	1428.14 (0.00)
Oats	-0.0111	3.6019	0.40	21.03	25391.24 (0.00)
Soybeans	-0.0321	8.3924	-0.75	9.55	3522.22 (0.00)
Soybean Meal	-0.0056	3.0539	-0.36	9.47	3307.93 (0.00)
Soybean Oil	0.0009	0.4575	-0.01	11.25	22023.20 (0.00)
Spring Wheat	-0.0077	6.9716	-0.74	14.54	24393.12 (0.00)
Sugar #11	-0.0022	0.1989	-0.13	8.38	2250.92 (0.00)
Wheat	-0.1053	7.4876	0.10	57.83	169022.5 (0.00)
Wheat #1 Cash	-0.1332	5.1916	-0.24	12.59	5193.38 (0.00)

Note: The test for the normality is performed using the Jarque-Bera statistic. The values in the parenthesis in the last column are the reported *p*-values.

distribution, implying leptokurticity.[6] According to the Jarque-Bera statistic, the null of normality is rejected for all series at the conventional significance levels.

IV. Empirical Results

As a preliminary step before applying the stable distribution to a time series, confirming whether or not a time series is stationary is an important issue. Non-stationarity, possibly due to inflation, will have ill-effects on the analysis. A distribution with non-stationarity might be too irregular to be described by the stable or even normal fitting, thus the finite sample behavior of the estimator cannot be precisely known. Therefore, checking for stationarity is necessary, and if any sign of non-stationarity is revealed, the data must be transferred or filtered to be stationary series.

The augmented Dickey-Fuller (1981) (ADF) test was performed on the series to check for the existence of a unit-root, and the results are displayed in Table 2. The ADF test results indicate that most level series have a unit-root process, indicating that they are non-stationary, except for broilers, large white eggs, and soybean oil. When the same test was applied to the first differenced series, price changes, the null of a unit-root was clearly rejected for all series at all conventional significance levels. The results of the ADF test for price changes are not displayed here, but they are obtainable from the author. Rejecting existence of unit-root process does not necessarily guarantee that the time series are stationary because the unit-root process is only an important source of non-stationarity. To confirm that the series of price changes are stationary, the Kwiatkowski, Phillips, Schmidt, and

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[6] Kurtosis measures the peakedness or flatness of a distribution, in other words the thickness of the tail of the distribution. The kurtosis of the normal distribution is 3. If the kurtosis exceeds 3, the distribution is peaked and it has a thicker tail relative to the normal; it is called leptokurtic. If the kurtosis is less than 3, the distribution is flat and it has a thinner tail relative to the normal; it is called platykurtic.

Shin (KPSS 1992) test was performed. The null of stationarity is not rejected for any of the series, implying that all price change series are stationary.

Starting values of the four parameters, α, β, δ and γ, of the stable distribution on the price changes are estimated, using the McCulloch's quantile method. These estimates are then used as initial values of the parameters in maximizing the log-likelihood function corresponding to Equation 2. To calculate the stable density of the price changes, the method suggested by Nolan (1997) is used. Further, to generate confidence intervals, a covariance matrix is calculated, following DuMouchel (1973).

TABLE 2. Results of the ADF and KPSS Tests for Daily Agricultural Cash Prices.

Variables	ADF Test (for Level Series)	KPSS Test (for Price Changes)	
		The Null 1	*The Null 2*
Broilers	-5.41**	0.01	0.01
Cocoa	-0.48	0.16	0.12
Coffee	-2.32	0.13	0.06
Corn	-0.46	0.05	0.03
Large White Eggs	-6.36**	0.04	0.02
Kansas City Wheat	-0.51	0.06	0.05
Oats	-0.55	0.08	0.06
Soybeans	-0.42	0.06	0.05
Soybean Meal	-0.45	0.03	0.03
Soybean Oil	-3.69**	0.12	0.03
Spring Wheat	-3.00	0.03	0.02
Sugar #11	-0.72	0.25	0.12
Wheat	-0.83	0.17	0.06
Wheat #1 Cash	-1.08	0.21	0.05

Note: The null hypothesis of the ADF test is that the time series has a unit-root process, indicating that the process is non-stationary. The null hypothesis of the KPSS test is stationarity of the series against a unit-root process. In the KPSS test, the null 1 denotes the stationarity hypothesis without drift, and the null 2 denotes the stationarity hypothesis with drift. The superscript ** denotes that the null hypothesis is rejected at the 5 percent significance level.

Estimates of the stable distribution parameters are reported in Table 3. The results show that none of the series correspond to the normal distribution. The $\hat{\alpha}$ values are away from 2.0 and they lie between 0.61 and 1.77, implying that the distributions are not normal and the series have thicker tails than those of the normal distribution. A $\hat{\alpha}$ value less than 2.0 implies that for the price changes, variance is not a proper measure of dispersion. The results match with the kurtosis and normality test results in Table 1. The values of $\hat{\alpha}$ for eggs and oats are less than 1.0, which indicates that the shapes of their distributions are different from those of other commodities and the traditional first moment, mean, may not be definable.[7]

TABLE 3. Estimated Parameters of the Stable Distribution: Fitting to Daily Agricultural Commodity Cash Price Changes.

Variables	Characteristic Exponent α	Skewness Parameter β	Scale Parameter γ	Location Parameter δ
Broilers	1.64 (0.067)	-0.01 (0.168)	0.52 (0.022)	0.01 (0.040)
Cocoa	1.77 (0.063)	0.34 (0.228)	13.98 (0.574)	-1.47 (1.103)
Coffee	1.26 (0.068)	0.00 (0.104)	1.54 (0.086)	-0.77 (0.115)
Corn	1.58 (0.071)	-0.06 (0.156)	1.99 (0.093)	0.99 (0.159)
Large White Eggs	0.61 (0.039)	0.22 (0.053)	0.09 (0.008)	-0.01 (0.004)
Kansas City Wheat	1.64 (0.070)	0.09 (0.178)	3.32 (0.149)	-0.12 (0.266)
Oats	0.62 (0.042)	-0.10 (0.057)	0.36 (0.036)	0.45 (0.168)
Soybeans	1.59 (0.072)	-0.01 (0.160)	4.30 (0.200)	0.16 (0.344)
Soybean Meal	1.52 (0.072)	0.08 (0.141)	1.48 (0.072)	-0.06 (0.118)
Soybean Oil	1.46 (0.035)	0.07 (0.064)	0.21 (0.003)	-0.01 (0.008)
Spring Wheat	1.42 (0.047)	0.00 (0.082)	2.95 (0.099)	0.03 (0.152)
Sugar #11	1.67 (0.069)	-0.05 (0.184)	0.10 (0.004)	0.004 (0.008)
Wheat	1.77 (0.076)	0.12 (0.278)	3.79 (0.186)	-0.12 (0.354)
Wheat #1 Cash	1.54 (0.084)	0.24 (0.169)	2.48 (0.138)	0.41 (0.233)

Note: The values in parentheses are estimates of the 95% confidence interval for the parameter estimates.

[7] We have checked white large eggs and oats for data errors, missing values, or excessive no change days due to thin markets. However, there is no bad sign for the series.

To check for consistency of the stable distribution in other frequencies of the price changes, the estimation was repeated for weekly frequency data (n=5) and bi-weekly frequency data (n=10). The values of $\hat{\alpha}$, which is the most important parameter in the stable distribution, are similar at all three time frequencies daily (n=1), weekly, and bi-weekly. The $\hat{\alpha}$ of the three different series are statistically indistinguishable within the 95 percent confidence interval, suggesting consistency of the stable distribution regardless of time frequency.

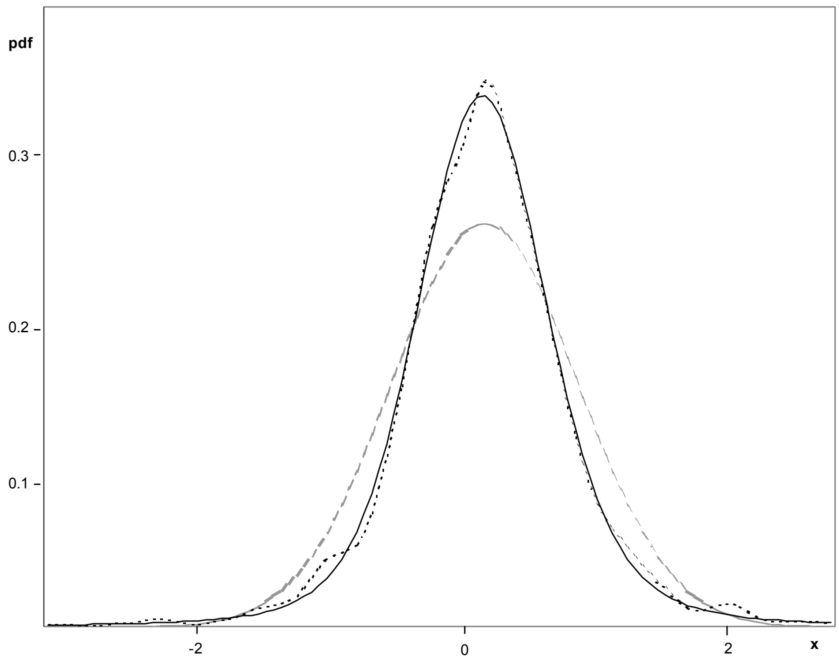
In order to see how effectively the stable distribution describes the data, the smoothed data density, the fitted stable density, and the fitted normal density are plotted together in a graph for each commodity. One such plot is displayed in Figure 1. The figure of Kansas City wheat was selected to be displayed here because it shows more clearly the three different densities

Table 4. Characteristic Exponent of the Stable Distribution: Fitting to Daily, Weekly, and Bi-Weekly Agricultural Cash Price Changes.

Variables	Characteristic Exponent α		
	Daily	Weekly	Bi-Weekly
Broilers	1.64 (0.067)	1.66 (0.148)	1.51 (0.216)
Cocoa	1.77 (0.063)	1.83 (0.131)	1.86 (0.173)
Coffee	1.26 (0.068)	1.11 (0.141)	1.06 (0.195)
Corn	1.58 (0.071)	1.56 (0.160)	1.52 (0.226)
Large White Eggs	0.61 (0.039)	0.61 (0.088)	0.60 (0.123)
Kansas City Wheat	1.64 (0.070)	1.68 (0.155)	1.72 (0.212)
Oats	0.62 (0.042)	0.65 (0.707)	0.70 (0.147)
Soybeans	1.59 (0.072)	1.70 (0.152)	1.82 (0.139)
Soybean Meal	1.52 (0.072)	1.56 (0.161)	1.70 (0.208)
Soybean Oil	1.46 (0.035)	1.45 (0.078)	1.42 (0.110)
Spring Wheat	1.42 (0.047)	1.38 (0.104)	1.43 (0.128)
Sugar #11	1.67 (0.069)	1.72 (0.149)	1.66 (0.219)
Wheat	1.77 (0.076)	1.86 (0.143)	1.87 (0.231)
Wheat #1 Cash	1.54 (0.084)	1.66 (0.183)	1.65 (0.260)

Note: The values of parenthesis are 95% confidence interval estimates for the estimated value of the parameters.

FIGURE 1. Plots of Data Density, Stable Fitted Density, and Normal Fitted Density: Daily Cash Price Changes of Kansas City Wheat.



Note: The solid line denotes the stable fitted density, the dotted line denotes the empirical smoothed data density, and the dotted grey line denotes the Normal fitted density.

than the other figures. In the figure, there are no clear multiple modes or gaps, which supports the premise that the data come from one of the stable distributions. The density plots admit two interpretations. First, none of the series seem to have a high degree of asymmetry or skewness. Second, the normal distribution does not effectively explain the leptokurticity of the series the stable distribution tracks the behavior of the agricultural cash prices better than the normal distribution can. The figures of the other commodities have the same interpretations, and they are obtainable from the author.

V. Conclusions

Studies have shown that commodity price series have two properties long-term dependence and leptokurticity. Leptokurticity can be effectively captured using the stable distribution model. The time series of major U.S. agricultural commodity cash prices have been analyzed to see fat-tails behaviors of the series and to test whether the behavior is captured by the stable distribution better than the normal distribution. Empirical results indicate evidence of leptokurtic behaviors, and the set of stable distributions explains the data more effectively than the normal distribution, which shows that agricultural cash price series do not bear out the Gaussian assumption.

Results also showed that most cash price changes have $\hat{\alpha}$ less than 2.0 but greater than 1.0, which suggests that variance can be undefined or infinite. Sample variances may be inappropriate measures of dispersion for the series. Therefore, in such cases, using variance in an economic analysis might lead to misleading results. If so, then alternative measures of dispersion are needed. Cornew, Town, and Crowson (1984) suggest using the scale parameter, γ, of the stable distribution when $\alpha<2$ so that the variance of the stable distribution is infinite. Even when $\alpha<2$, the scale parameter, γ, is finite and it does a good job in measuring dispersion of the distribution. However, the parameter γ does not exactly share the same properties with the variance (Fielitz and Roselle, 1983). This suggests that whether or not the parameter can be used as a suitable alternative for the variance when $\alpha<2$ depends upon further discovery of the properties of the scale parameter, γ.

This study provides empirical contributions at least in three areas. First, finding that leptokurticity is explained poorly by the traditional normal distribution and instead explained well by the stable distribution might provide new insights in the economic analyses of agricultural financial and commodity markets. Second, under the stable distribution, variance may be undefinable and it may not effectively describe the dispersion of the data in analysis. This raises the necessity of finding alternatives to the traditional second moments, variance. Third, the comparison of efficiency between the two competing distributional models (the normal and stable distribution) in capturing the behaviors of financial and economic time series has been elusive in the previous studies which used the stable distribution for financial data. This study performs the comparison by plotting the fitted stable density and fitted normal density and by comparing them with the smoothed data density.

This study suggests imposing a more correct distributional setting by comparing the performance of the normal distribution with the stable distribution in measuring the behaviors of cash price changes. The empirical results indicate that time processes of price changes do not follow the normal distribution. Therefore, optimal hedge ratios estimated by using the normality assumption and variance will be different from hedge ratios under the relaxation of the narrow normality assumption, suggesting that imposing normality causes a bias in optimal hedge ratios. Estimates of hedging performance and hedger's surplus under the wrong distributional model restricted by the normality might also be biased.

Farmers and agribusiness managers expect well-behaved equilibrium price in the next period, but price in the next period may not follow well-behaved process. Agricultural commodity prices are affected by many sources. Among them, shocks to supply are major sources, and their distributions are not regular, implying that there are substantial chances of abrupt, large changes in their processes. In practice, mild price variation may not give much economic effect to producers. Instead, abrupt, large changes might cause an unexpected damage to them. The most important reason for purchasing insurance or hedging in futures markets is to avoid risks from unexpected, catastrophic large changes in prices. Thus, it is important to describe the tail behaviors of data effectively to see possible risks from such extreme events and to incorporate information of the risks correctly into a risk management tool. In the normal distributional setting, such extreme events do not receive much weight. Normal distribution screens out the outliers so that it is not appropriate to describe tail behavior of data. However, by using the stable distribution and scale parameter, γ, we can capture more efficiently the tails behavior of price distribution.

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