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A LONG MEMORY CONDITIONAL VARIANCE MODEL FOR INTERNATIONAL GRAIN MARKETS*

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Keywords
international grain markets, stochastic volatility, FIGARCH, non-normality

Abstract
The study explores a long memory conditional volatility model on international grain markets, demonstrating importance of modeling both temporal effects of volatility and long memory process. This study adopts six different volatility models, nested in an ARMA(p,q)-FIGARCH(P,D,Q), to capture dependence of grain cash price volatility and compares the performance of the six models. It also visits a related question about non-normal behaviors of grain prices and adopts the *student-t density* intended to account for fat-tailed properties of the data. We find suitability of the FIGARCH type models under the student-t distribution and competitiveness of the parsimonious FIGARCH(1,d,0) for modeling long memory volatility.

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1. Introduction

A substantial body of studies has shown long memory behaviors in the mean and volatility processes of commodity prices and financial assets.[1] In this context, efforts have been made to specify both a long memory parameter and temporal effect variables in a framework, and Baillie, Bollerslev, and Mikkelsen (1996) among them introduced the fractionally integrated GARCH (FIGARCH) process. The model has been successful in capturing the slow movements of volatility clustering of stock returns. For fractionally integrated processes, the effect of a shock decays much more slowly than for a process integrated of order zero, but it does not exhibit infinite dependence. In this sense, fractional integration captures long memory dynamics more parsimoniously than a non-integrated process and it is more realistic than the integer-integrated models (Jin, Elder, and Koo, 2006).

When a series exhibits long memory, there is temporal dependence in the first and second moment even between distant observations. The dependency provides a predictable component in series dynamics, which would be another source for increasing forecasting accuracy and enhancing performance of risk management. Therefore, analyzing whether the long memory conditional variance model captures volatility behaviors of commodity or asset prices better than the conventional GARCH type model is an important task for more efficient forecasting and risk management models.

This study applies a long memory volatility model to international grain markets. Studies have examined the characteristics of volatility in individual markets trading foreign currencies, stocks, bonds, and commodities, but comparatively little attention has been given to the volatility of grain cash prices. The following studies are found to be related to grain cash or futures price dynamics. Barkoulas, Labys, and Onochie (1997) analyzed returns on monthly price levels of twenty-one commodities over the thirty-year period, and they found evidence of fractional integration for six out of all series. Crato and Ray (2000) found evidence of long memory in the volatilities of mean-corrected com-

[1] Baillie (1996) provides an extensive review of the major econometric works on long memory and fractional integration processes and the applications of previous studies in economics and finance.

modity futures returns. Jin and Frechette (2004a) discovered suitability of the fractional integration for the volatility of commodity futures returns. Jin and Frechette (2004b) tested for long memory in daily and weekly agricultural cash price returns, using *the modified rescaled range* (R/S) test and a corrected *t*-test and found evidence of long memory in more than half of the agricultural commodities analyzed. Elder and Jin (2007) reexamined commodity futures for evidence of fractional integration, utilizing two *wavelets-based estimators*. They found an evidence of long memory, in the form of anti-persistence, in about half of agricultural commodity futures, while they found little evidence of long memory in metal futures. However, no study has been performed regarding fractional integration behavior of grain cash price volatility and this study explores the issue.

The purpose of this study is to find a suitable volatility model for grain prices. We compare the performances of six competing volatility models, nested in an ARMA(p,q)-FIGARCH(P,D,Q) process, in capturing dependence of grain cash price volatility. The comparison will provide another layer of empirical supports for using the long memory volatility models over the conventional models. This study also emphasizes a related question about non-normal behaviors of grain prices since empirical studies have shown that commodity and financial data have distributions with fatter tails and higher peaks than the normal distribution, known as leptokurticity.[2] This paper adopts the *student-t density* intended to account for fat-tailed properties of the data, and it is compared to the case assuming conditional normality. A normality test is carried out for the return series of the grain prices. When the series exhibit fat-tails behaviors, the student-*t* density will be applied to the estimation of the volatility models. Lastly, this study compares the performances of the FIGARCH(1,d,1) and FIGARCH(1,d,0) models. Some studies suggest that the FIGARCH(1,d,0) model is parsimonious, but performance of the model is comparable to that of the FIGARCH(1,d,1) model (e.g., Baillie *at al*., 1996; and Tse, 1998). The comparison would illustrate a more appropriate long memory volatility model for grain prices.

The commodities of interest are three grains traded on international markets: U.S. wheat, corn, and soybeans. These are important commodities in

2 See, for example, Fielitz and Rozelle, 1983; Hall, Brorsen, Irwin, 1989; Gribbin, Harris, and Lau, 1992; and McCulloch, 1996.

the world grain market. The market has become more competitive due to growth and enhanced productivity of grain exporting countries, which impacts U.S. market shares and export prices because U.S. exporters are losing their capability of controlling quantity and price (refer, e.g., Pick and Park, 1991; Love and Murniningtyas, 1992; and Carter, MacLaren, and Yilmaz, 1999). For a completed comparison of the competitive models[3], only the three grains are selected. This study does not attempt to analyze a large variety of commodities because this would involve a myriad of estimations and presentations of the results. Rather, we focus on a fewer number of, but the most important, grains.

The remainder of the paper is organized as follows: The second section provides a discussion about the GARCH and FIGARCH models. The third section details the data used in this study and the fourth section presents the empirical results. In the fifth section, the normality assumption for the price series is relaxed and the results are compared to the case assuming the normal distribution. The last section summarizes and concludes.

2. The GARCH and FIGARCH Processes

The GARCH type conditional variance models have been extensively used in the studies for price dynamics and risk control (e.g., Baillie and Myers, 1991; Kroner and Sultan, 1993; and Haigh and Holt, 2002). However, the GARCH processes have the limitation on capturing long-term dependence because persistence in these models decays relatively fast, while studies reveal that time processes of commodity and financial assets prices have long-term dependency even between observations at long lags. To remedy this shortcoming, Engle and Bollerslev (1986) introduced the Integrated GARCH (IGARCH) model, but the process exhibits unrealistic infinite persistence. This suggests that the knife-edging distinction of either zero (corresponding to GARCH models) or integer integration (corresponding to IGARCH models) of stochastic volatility series may not provide a relevant specification for explaining long-term volatility behav-

[3] For each commodity, six conditional variance models are specified and compared: GARCH(1,1)-normal, GARCH(1,1)-student-*t*, FIGARCH(1,*d*,1)-normal, FIGARCH (1,*d*,1)-student-*t*, FIGARCH(1,*d*,0)-normal, and FIGARCH(1,*d*,0)-student-*t*.

iors, whereas fractional integration might be fitted to such a purpose.

Stochastic form of volatility models have been extensively used and the most popular is the GARCH process. Following Bollerslev (1986), the GARCH(p,q) model is given by the following two equations:

(1) $y_t = \mu + E_{t-1}[y_t] + \varepsilon_t$,

(2) $\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2$,

where the series $\{\varepsilon_t\}$is the deviation from the conditional mean for some other process $\{y_t\}$, $E_{t-1}[.]$ is the mathematical expectation, conditional on the information set at time *t-1*, σ_t^2 is the conditional variance of ε_t, L denoting the back-shift operator, and $\alpha(L) \equiv \alpha_1 L + \alpha_2 L^2 + \cdots + \alpha_q L^q$, $\beta(L) \equiv \beta_1 L + \beta_2 L^2 + \cdots + \beta_p L^p$. If the roots of $[1 - \alpha(L) - \beta(L)]$ and $[1 - \beta(L)]$ lie outside the unit circle, then $\{\varepsilon_t^2\}$ exhibits stability and covariance stationarity.

The GARCH type models have been successful in modeling stochastic volatility dynamics and have been applied to numerous empirical studies. However, the models face limitations in capturing long-term volatility behaviors of time series. Studies have reported a geometric rate of decay in volatility shocks (e.g., Ding, Granger, and Engle, 1993), which is neither an exponential rate of decay in propagation of shocks in an I(0) process nor a permanent dependence of shocks in an I(1) process. This suggests that both the GARCH and IGARCH models are not effective in capturing the persistence. The distinction between the integer orders is therefore too restrictive for modeling low frequency volatility behaviors. Fractional integration is more adequate, in which shocks dampen more slowly than GARCH shocks, but they are not permanent.[4]

Following Baillie *at al*. (1996), if we expand the GARCH(p,q) models by incorporating the fractional difference operator, we obtain the FIGARCH (p,d,q) models as follows:

(3) $\varnothing(L)(1-L)^d \varepsilon_t^2 = \omega + [1 - \beta(L)]\, v_t$,

[4] Under the integer integration, time series are usually presumed to be integrated of order zero or one. When the order of integration is zero, the process is stationary, and effects of a shock decay geometrically. When the order of integration is one, the process is said to have a unit-root, and effects of a shock persist into the infinite future.

where $\varnothing(L) \equiv [1-\alpha(L)-\beta(L)](1-L)^{-1}$, $v_t \equiv \varepsilon_t^2 - \sigma_t^2$, d is the fractional integration operator which is typically between 0 and 1, and all the roots of $\varnothing(L)$ and $[1-\beta(L)]$ lie outside the unit circle. If we rewrite the process as an infinite-order ARCH process, the conditional variance of ε_t is given by

$$\text{(4)} \qquad \sigma_t^2 = \omega[1-\beta(L)]^{-1} + \{1 - [1-\beta(L)]^{-1} \varnothing(L)(1-L)^d\} \varepsilon_t^2 .$$

For the FIGARCH process in Equation (3) to be well-defined and the conditional variance to be positive almost surely for all t, all the coefficients in the infinite ARCH representation in Equation (4) must be non-negative. Using a direct extension of the proofs for the IGARCH case, Baillie *at al* (1996) show that the FIGARCH processes are strictly stationary and ergodic for $0 \le d \le 1$.[5]

The FIGARCH model nests the GARCH model when $d = 0$ and the IGARCH model when $d = 1$. For the covariance-stationary FIGARCH(p,d,q) model with $d = 0$, from a forecasting perspective, shocks to the conditional variance die out at a fast exponential rate, whereas when $d = 1$, shocks to the conditional variance persist indefinitely. In contrast, for the FIGARCH(p,d,q) model with $0 < d < 1$, shocks decay at a slow hyperbolic rate. Thus, although the cumulative impulse response function converges to zero, the fractional differencing parameter provides important information regarding the pattern and speed with which shocks to the volatility process are propagated. The value of the fractional differencing parameter depends therefore on the decay rate of a shock to conditional volatility. For values of $d > 1$, the conditional variance process is unrealistically explosive and the cumulative impulse response is undefined.

In most practical applications, relatively simple first-order models have been found to provide good representations of the conditional variance processes. That is, low-order GARCH models typically outperform high-order ARCH models. For example, McCurdy and Morgan (1988) and Hsieh (1989) found that the GARCH(1,1) model fits the second moments of currency prices better than high-order ARCH models. We therefore use the GARCH(1,1) as our basis model of conditional variance as follows:

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5 See pp 8-9 in Baillie, Bollerslev, and Mikkelsen (1996) for details on the debate concerning stationarity of the FIGARCH(p,d,q) class of processes.

(5) $$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 .$$

If we express the corresponding FIGARCH(1,*d*,0) process as an infinite-order ARCH representation, we obtain[6]

(6) $$\sigma_t^2 = \omega + \beta_1 \sigma_{t-1}^2 + [1 - \beta_1 L - (1-L)^d]\, \varepsilon_t^2 ,$$

and the corresponding FIGARCH(1,*d*,1) process is

(7) $$\sigma_t^2 = \omega + \beta_1 \sigma_{t-1}^2 + [1 - \beta_1 L - (1 - \varnothing_1 L)(1-L)^d]\, \varepsilon_t^2 .$$

3. Data

The three grains of interest are U.S. No.2 Hard Red Winter Ordinary wheat, U.S. No.3 yellow maize/corn, and U.S. No.2 yellow soybeans. The data on wheat and corn came from the *International Grains Council* in London, the United Kingdom, and the data on soybeans came form the *Oil World* in Hamburg, Germany. The data could be retrieved from the *UNCTAD Commodity Price Bulletin*. All of the series stem from spot or physical markets and can be termed cash prices. These markets function competitively, and none of the price series represent secondary or trade unit value prices. The series are monthly average prices per metric ton and are denoted in U.S. dollars. Monthly frequency represents a series in which both short-term and long-term memory can be observed without much noise. The monthly averages are calculated from daily quotations, except for wheat prices which are calculated from weekly quotations.

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6 The process can be rewritten as an ARMA(1,1) process in ε_t^2, $(1 - \varnothing_1 L)\varepsilon_t^2 = \omega + (1 - \beta_1 L)\, v_t$, where $\varnothing_1 \equiv \alpha_1 + \beta_1$. When $\alpha_1 + \beta_1$ is equal to one, the IGARCH(1,1) process occurs as follows: $(1-L)\varepsilon_t^2 = \omega + (1 - \beta_1 L)\, v_t$. Then, the corresponding FIGARCH(1,*d*,0) model is $(1-L)^d \varepsilon_t^2 = \omega + (1 - \beta_1 L)\, v_t$.

Original data set starts from January 1960. Remind that we experienced the so-called world food crisis in the 1970s and the Russian wheat shock in year 1973. Grain export prices roughly doubled in 1973 through 1974, largely as a consequence of global macroeconomic imbalance. The international prices of almost all commodities and raw materials increased sharply during this period, including petroleum, bauxite, copper, and food and farm commodities. The gravity of the situation was captured in food sector more dramatically due to the Russian wheat shock. For more details, refer, for example, Hopkins and Puchala (1978) and Paarlberg (1999). Therefore, one can easily expect a structural change in the international grain price dynamics (mean and volatility processes). If we specify the both periods before and after the early 1970s in a framework, it might produce misleading estimates of ARCH, GARCH, and fractional integration parameters. A possible remedy for this would be estimation of the models for two different periods: one for the time period before early 1970s and the other for the time period after early 1970s. For our sample data, the time span before the 1970s is relatively short and therefore, we will utilize sample data after the 1970s.

For a visual inspection of existence of a structural change (around early 1970s), we plotted the grain price series, which are displayed in Figures 1 through 3. The figures show distinctly different movements of the series before and after the year 1973. In the early period of the sample when price level was low, volatility was relatively weak for all three commodities. The post-1973 observations have higher price levels and more volatile movements than those in the early sample. Using the visual examination as a guide, we removed the sample from January 1960 through 1974, so the data used in this analysis run from January 1975 through December 2002. Note that due to the access limitation, data after January 2003 could not be used in this study. Summary statistics for the grain prices are presented in Table 1.

It is immediately evident from Figures 1 through 3 that the raw grain cash price series, *pt*, is nonstationary. The Augmented Dickey-Fuller (ADF) test was performed on the sample period to check for the existence of a unit-root, and the null of a unit-root was not rejected for all series at the conventional significance levels. Following standard practice, we shall therefore concentrate on modeling the monthly returns; i.e., $y_t \equiv \log(p_t/\, p_{t-1})$. The ADF test was performed again for the return series, and results indicate that the null of a unit-root was clearly rejected for all return series at all conventional significance levels.

FIGURE 1. Cash Prices of U.S. Wheat, Hard Red Winter Ordinary.

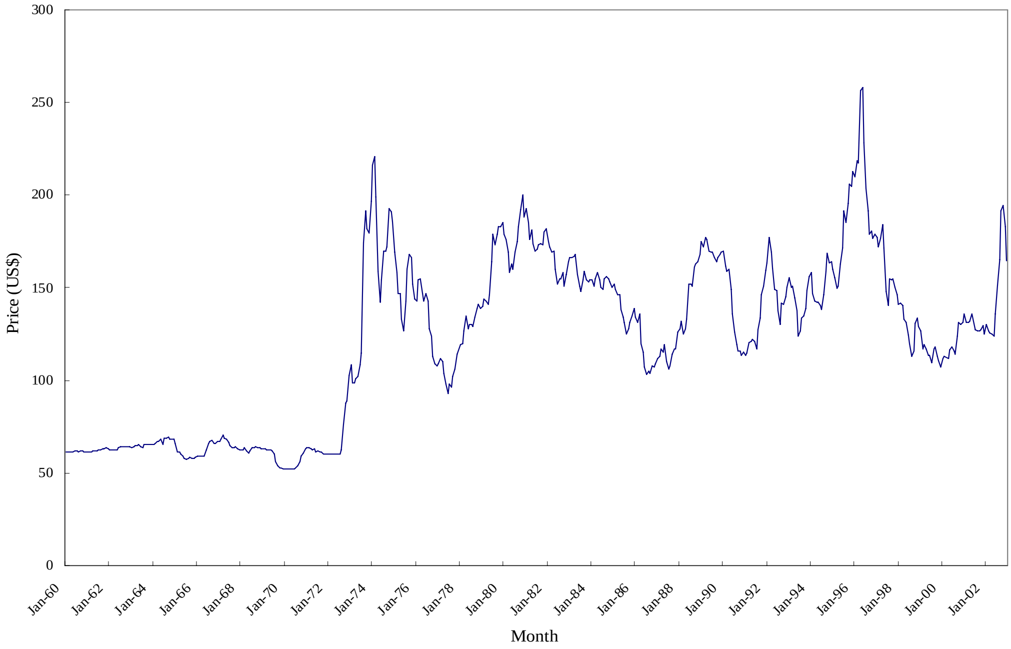

FIGURE 2. Cash Prices of U.S. Maize/Corn, No.3 Yellow.

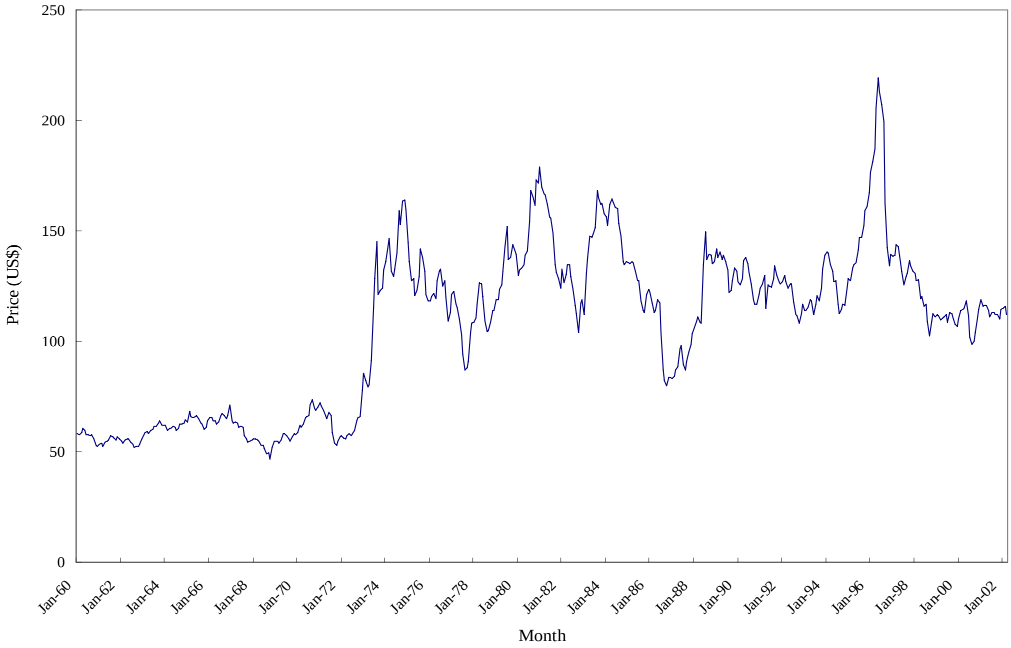

FIGURE 3. Cash Prices of U.S. Soybeans, No.2 Yellow.

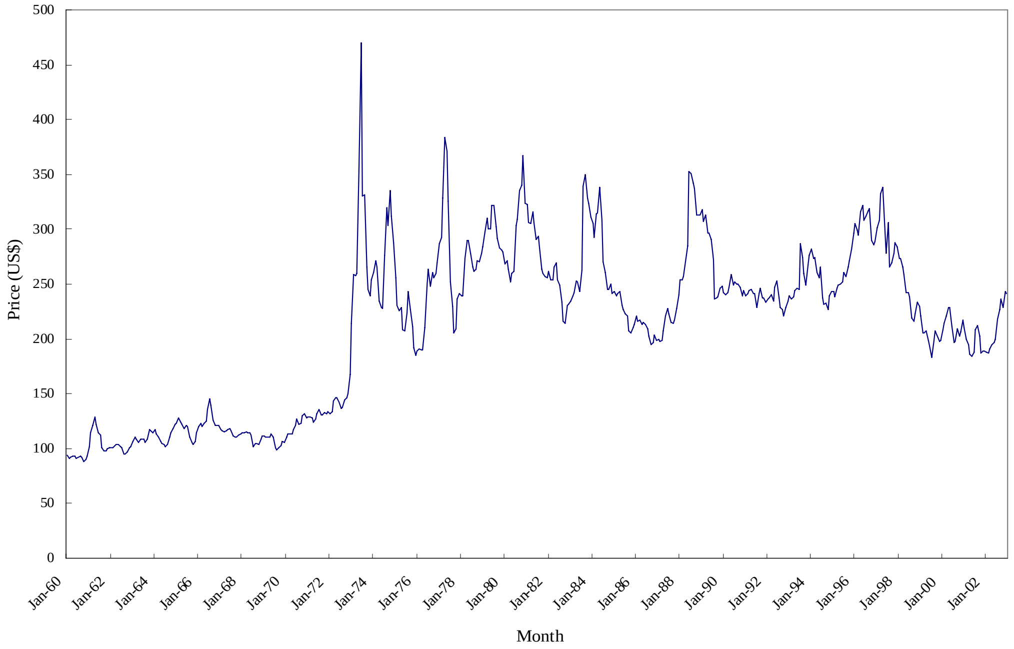

FIGURE 4. Returns Series of U.S. Wheat Cash Prices, Hard Red Winter Ordinary.

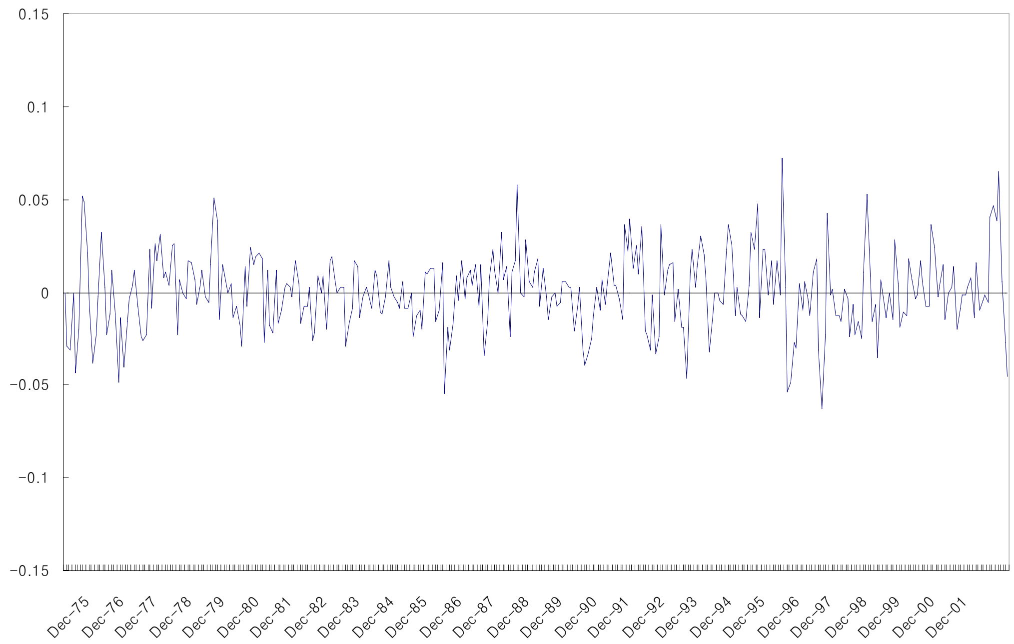

FIGURE 5. Return Series of U.S. Maize/Corn Cash Prices, No.3 Yellow.

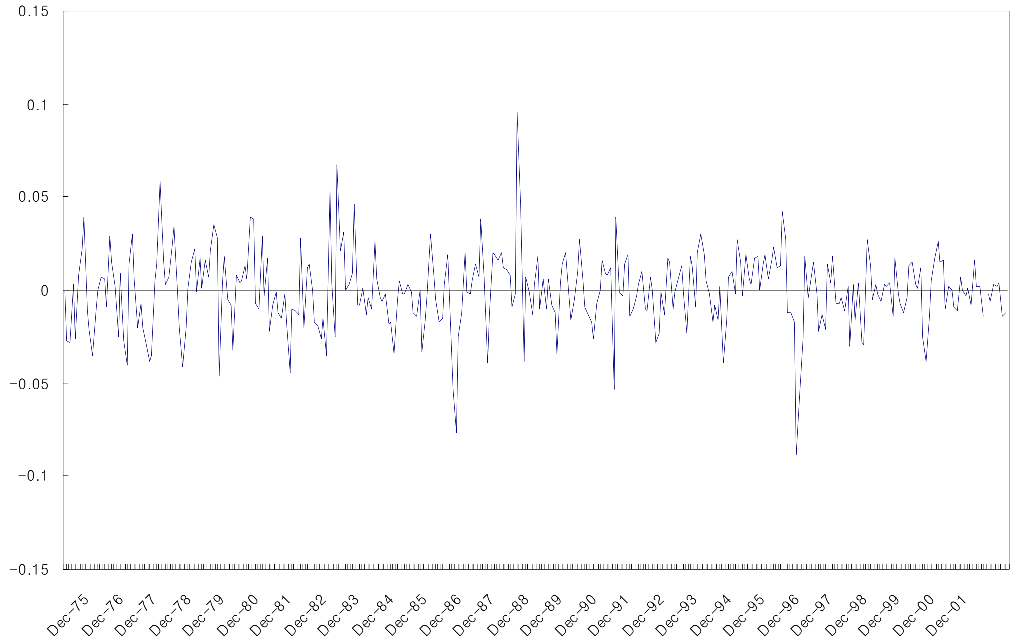

FIGURE 6. Return Series of U.S. Soybeans Cash Prices, No.2 Yellow.

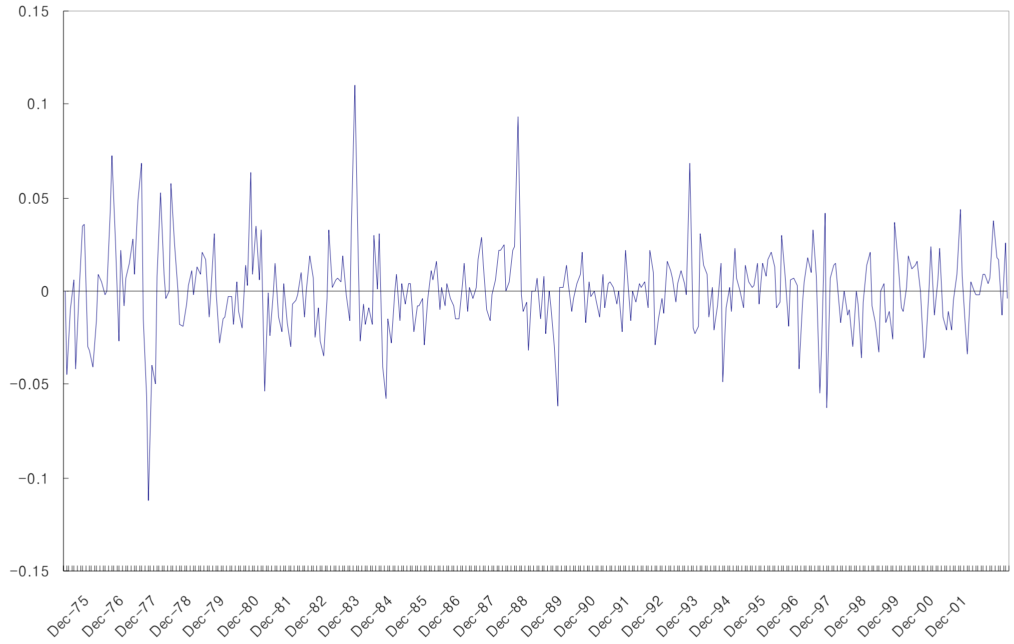

TABLE 1. Descriptive Statistics and Normality Test Results for Cash Price Series.

Statistics	U.S. Wheat	U.S. Corn	U.S. Soybeans
Mean	146.34	127.40	252.55
Standard Deviation	26.89	21.64	40.93
Maximum	258.10	219.40	384.00
Minimum	93.00	80.00	183.00
Skewness	0.661	0.980	0.552
Kurtosis	3.931	5.311	2.798
Normality Test	36.64 (0.000)	125.18 (0.000)	17.63 (0.000)

Notes: The normality test was completed using the Jarque-Bera statistic, where the null hypothesis is the normal distribution and the values in the parenthesis are *p*-values.

TABLE 2. Normality Tests for Filtered Return Series.

Return Series	Statistics	Wheat	Corn	Soybeans
Mean-corrected Returns	b_3	0.201	-0.045	0.236
	b_4	3.791	5.509	6.979
	N	10.97**	85.61**	224.17**
ARMA(1,0) filtered Returns	AIC	847.84	815.03	801.49
	b_3	0.215	0.187	0.190
	b_4	3.854	6.111	6.301
	N	12.80**	133.42**	154.13**
ARFIMA(1,d,0) filtered Returns	AIC	850.18	818.22	801.12
	b_3	0.247	0.163	0.457
	b_4	3.994	5.839	6.405
	N	17.24**	110.97**	173.58**

Notes: b_3 and b_4 denote the sample skewness and kurtosis, respectively, for the filtered return series. N denotes the Jarque-Bera normality test statistic, and the superscript** denotes rejection of the null hypothesis of normality at the 5 percent significance level.

To provide additional evidence, we also utilized the Kwiatkowski, Phillips, Schmidt, and Shin (1992) (KPSS) test, which has a null of stationarity. Again, the evidence suggests that the data are stationary, as the null of stationarity was not rejected for all commodities at the 5 percent significance level.[7] The results

[7] There are two different tests in the KPSS. The first tests the null hypothesis of sta-

of the ADF and KPSS tests are not reported here, but they are available from the authors upon request. The return series were also plotted, and they clearly indicate the occurrence of tranquil and volatile periods.

Subsequently, departure from the normality was assessed, focusing on the higher order of moments of the return series, the skewness and kurtosis. The results are presented in Table 2. To increase the generality of the test, we test three different return series filtered by different mean processes: mean-corrected, ARMA(1,0), and ARFIMA(1,*d*,0) processes. The model specifications of the ARMA and ARFIMA conditional mean processes were based on Akaike Information Criterion (AIC), and the mean models were estimated by conditional Gaussian Maximum Likelihood. All kurtosis coefficients are substantially larger than the value corresponding to the normal distribution, suggesting that the empirical distributions of the returns deviate from the normality in that they exhibit heavy tails. Normality test results by the Jarque-Bera statistic clearly reject the null hypothesis of normality for all filtered return series. Those results suggest that a student-*t* type distributional assumption would be better fitted to movements of the return series than the normality assumption.

4. Empirical Results

This study investigates a relevant framework of conditional variance model through comparison among different specifications of the FIGARCH(*p,d,q*) model under both the normal distribution and the student-*t* distribution. Therefore, when we interpret the results, we focus on examining which conditional variance model provides the most parsimonious representation and best performance in capturing dependence in the volatility dynamics of the grain prices.

The GARCH(1,1), FIGARCH(1,*d*,1), and FIGARCH(1,*d*,0) models are estimated for the grain price returns under the normality assumption. For all conditional variance models, we use the same conditional mean model, ARMA (1,0). The FIGARCH models are estimated using Bollerslev and Wooldridge's

tionarity without drift and the second tests the null of stationarity with drift. The both null hypotheses were not rejected at the 5 percent significance level.

TABLE 3. Results of GARCH(1,1), FIGARCH(1,*d*,1), and FIGARCH(1,*d*,0) Estimations for U.S. Wheat Return Series.

Variables and Statistics	GARCH(1,1)		FIGARCH(1,*d*,0)		FIGARCH(1,*d*,1)	
	Normal	Student-*t*	Normal	Student-*t*	Normal	Student-*t*
μ (Constant for Mean)	0.0001	-0.0002	-0.0001	-0.0002	-0.0001	-0.0002
	(0.01)	(-0.21)	(-0.09)	(-0.18)	(-0.10)	(-0.21)
α_1 (AR(1))	0.285	0.287	0.280	0.281	0.279	0.281
	(4.85)	(5.49)	(5.11)	(5.45)	(5.24)	(5.45)
ω (Constant for Variance)	0.923	0.142	0.723	0.709	0.714	0.709
	(1.04)	(0.80)	(1.12)	(0.88)	(2.16)	(1.55)
β_1(ARCH1)	0.076	0.039	0.049	0.001	-	-
	(1.34)	(1.27)	(0.16)	(0.01)		
α_1 (GARCH1)	0.664	0.919	0.292	0.247	0.254	0.247
	(2.36)	(12.78)	(0.77)	(0.49)	(2.20)	(1.45)
d (Fractional Integration)	0 (Restricted)	0 (Restricted)	0.277	0.290	0.285	0.290
			(1.97)	(1.57)	(2.64)	(1.96)
Student DF	-	8.155**	-	7.946**	-	7.946**
Log Likelihood	804.371	808.69	803.87	808.84	803.85	808.84
b_3	0.212	0.167	0.124	0.121	0.116	0.120
b_4	3.981**	3.989**	4.148**	4.176**	4.172**	4.176**
AIC	-5.075	-5.096	-5.065	-5.091	-5.072	-5.097
Q(20)	17.674	17.486	18.181	18.178	18.232	18.178
Q(100)	82.828	86.207	89.596	89.698	90.301	89.697
$Q^2(20)$	33.786**	29.881**	29.361	25.312	25.718	25.312
$Q^2(100)$	109.68	111.636	114.575	114.024	114.822	114.024
$ARCH_{10}$	1.566	1.538	1.194	1.167	1.167	1.171
EGARCH Test	0.640	0.781	0.531	0.656	0.558	0.656

Notes: Values in parentheses are *t*-statistics. b_3 and b_4 denote the sample skewness and kurtosis for the residuals, where the null hypotheses are no excess skewness and no excess kurtosis. The Q(*k*) and $Q^2(k)$ are the Box-Pierce Q-statistic at lag *k* for residual and squared residual series, respectively. The null hypothesis is no autocorrelation up to lag *k*. $ARCH_{10}$ means the test of autoregressive conditional heteroskedasticity up to lag 10. The null hypothesis of ARCH test is no ARCH effects up to lag *k*. The null hypothesis of EGARCH test is that the three effects (sign bias, negative size bias, and positive size bias) are not statistically significant. The superscript ** denotes rejection of the null hypothesis in each test at the 5 percent significance level.

TABLE 4. Results of GARCH(1,1), FIGARCH(1,*d*,1), and FIGARCH(1,*d*,0) Estimations for U.S. Corn Return Series.

Variables and Statistics	GARCH(1,1)		FIGARCH(1,*d*,0)		FIGARCH(1,*d*,1)	
	Normal	Student-*t*	Normal	Student-*t*	Normal	Student-*t*
μ (Constant for Mean)	-0.0001	-0.00008	0.0001	-0.00006	-0.0007	0.0003
	(-0.07)	(-0.05)	(0.10)	(-0.04)	(-0.83)	(0.30)
α_1 (AR(1))	0.299	0.318	0.286	0.318	0.245	0.322
	(5.61)	(6.44)	(4.80)	(6.32)	(6.56)	(7.21)
ω (Constant for Variance)	0.974	0.968	0.169	0.992	0.625	0.634
	(0.57)	(0.48)	(4.27)	(0.37)	(1.33)	(0.72)
α_1(ARCH1)	0.025	0.032	0.918	0.740	-	-
	(1.01)	(0.73)	(29.14)	(1.21)		
β_1 (GARCH1)	0.731	0.724	0.974	0.711	0.298	0.253
	(1.64)	(1.35)	(67.37)	(1.04)	(3.97)	(1.71)
d (Fractional Integration)	0 (Restricted)	0 (Restricted)	0.127	0.007	0.374	0.365
			(2.17)	(0.04)	(5.37)	(2.56)
Student DF	-	4.904**	-	4.895**	-	5.670**
Log Likelihood	784.72	800.24	788.66	800.27	756.32	784.36
b_3	0.184	0.197	0.331**	0.195	0.152	0.206
b_4	6.112**	6.167**	5.761**	6.188**	7.074**	7.641**
AIC	-4.951	-5.042	-4.969	-5.036	-4.770	-4.942
Q(20)	11.475	11.672	13.329	11.672	11.143	10.516
Q(100)	104.157	102.884	113.914	102.866	100.225	93.447
Q^2(20)	10.727**	10.577	9.624	10.378	9.509	8.795
Q^2(100)	111.266	110.638	98.478	110.098	112.416	111.245
$ARCH_{10}$	0.328	0.372	0.158	0.317	0.568	0.550
EGARCH Test	7.359**	7.007**	2.587	6.728	4.898	5.455

Notes: Values in parentheses are *t*-statistics. b_3 and b_4 denote the sample skewness and kurtosis for the residuals, where the null hypotheses are no excess skewness and no excess kurtosis. The Q(*k*) and $Q^2(k)$ are the Box-Pierce Q-statistic at lag *k* for residual and squared residual series, respectively. The null hypothesis is no autocorrelation up to lag *k*. $ARCH_{10}$ means the test of autoregressive conditional heteroskedasticity up to lag 10. The null hypothesis of ARCH test is no ARCH effects up to lag k. The null hypothesis of EGARCH test is that the three effects (sign bias, negative size bias, and positive size bias) are not statistically significant. The superscript ** denotes rejection of the null hypothesis in each test at the 5 percent significance level.

Table 5. Results of GARCH(1,1), FIGARCH(1,*d*,1), and FIGARCH(1,*d*,0) Estimations for U.S. Soybeans Return Series.

Variables and Statistics	GARCH(1,1)		FIGARCH(1,*d*,0)		FIGARCH(1,*d*,1)	
	Normal	Student-t	Normal	Student-t	Normal	Student-t
μ (Constant for Mean)	-0.0006	-0.0002	-	-0.0002	-0.0011	-0.0001
	(-0.41)	(-0.18)		(-0.17)	(-0.84)	(-0.09)
α_1(AR(1))	0.246	0.199	-	0.201	0.221	0.202
	(3.54)	(3.50)		(3.40)	(3.40)	(3.66)
ω (Constant for Variance)	0.844	0.873	-	0.970	0.680	0.666
	(3.01)	(1.41)		(0.99)	(4.11)	(1.50)
1(ARCH1)	0.155	0.127	-	0.811	-	-
	(3.91)	(1.73)		(4.45)		
α_1(GARCH1)	0.675	0.722	-	0.687	0.111	0.226
	(7.71)	(4.57)		(3.07)	(1.05)	(1.34)
d(Fractional Integration)	0 (Restricted)	0 (Restricted)	-	0.029	0.319	0.334
				(0.14)	(3.59)	(1.85)
Student DF	-	3.639**	-	3.613**	-	4.221**
Log Likelihood	761.22	781.19	-	781.30	753.49	778.87
b_3	0.568**	0.581**	-	0.561**	0.745**	0.811**
b_4	5.092**	6.144**	-	5.993**	6.892**	7.303**
AIC	-4.801	-4.922	-	-4.916	-4.752	-4.907
Q(20)	10.922	12.576	-	12.435	10.193	10.912
Q(100)	71.428	74.534	-	74.537	73.996	74.022
$Q^2(20)$	15.252	15.321	-	15.288	13.390	14.188
$Q^2(100)$	131.925**	131.552**	-	130.01**	115.952	123.862**
$ARCH_{10}$	0.395	0.389	-	0.374	0.295	0.346
EGARCH Test	2.224	3.130	-	2.727	1.646	2.422

Notes: Values in parentheses are *t*-statistics. b_3 and b_4 denote the sample skewness and kurtosis for the residuals, where the null hypotheses are no excess skewness and no excess kurtosis. The Q(k) and $Q^2(k)$ are the Box-Pierce Q-statistic at lag k for residual and squared residual series, respectively. The null hypothesis is no autocorrelation up to lag k. $ARCH_{10}$ means the test of autoregressive conditional heteroskedasticity up to lag 10. The null hypothesis of ARCH test is no ARCH effects up to lag k. The null hypothesis of EGARCH test is that the three effects (sign bias, negative size bias, and positive size bias) are not statistically significant. The superscript ** denotes rejection of the null hypothesis in each test at the 5 percent significance level.

(1992) quasi-maximum likelihood estimator (QMLE), which allows for asymptotically valid inference when the standardized innovations are not normally distributed.[8] For the GARCH(1,1) specification, the differencing parameter, *d*, is restricted to zero.

The estimation results are displayed in Tables 3 through 5. The log-likelihood statistic and AIC, along with the skewness (b_3) and the kurtosis (b_4) for the standardized residuals of the estimated models, are reported. In what follow, Ljung and Box Q-statistics provide information about serial correlation. The Q-statistics test for remaining autocorrelation in residuals and squared residuals in the variance equations. There is one Q-statistic for each lag *k*, and it is distributed as $\chi^2(k)$ under the null hypothesis of no autocorrelation up to lag *k*. If the variance equation is correctly specified, then all Q-statistics should be small and statistically insignificant. Finally, the test statistics of ARCH and EGARCH effects in the residuals are presented in the last two row s.[9] The EGARCH test is a joint test, proposed by Engle and Ng (1993), for sign bias, negative size bias, and positive size bias. The *sign bias test* investigates the impact of positive and negative return shocks on volatility not predicted by the model under construction. The negative (positive) *size bias test* focuses on the different effects that large and small negative (positive) return shocks have on volatility not predicted by the volatility model. It is a Lagrange Multiplier (LM) test for adding the three effect variables in the conditional variance equation. If the volatility model being specified is correct, then the null hypothesis says that the three effects are not statistically significant.

We also investigated higher orders of GARCH and FIGARCH models; for example, GARCH(2,1), GARCH(2,2), GARCH(3,1), FIGARCH(2,*d*,1), FIGARCH(2,*d*,2), and FIGARCH(3,*d*,1). Based on AIC and Log-Likelihood statistics, there is no evidence that any higher order is required. For all different FIGARCH processes, it was found that regardless of the specification adopted,

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8 Bollerslev and Mikkelsen (1996) used a simulation to show that the QMLE performs well in estimating FIGARCH models, compared to alternative methods.

9 The null hypothesis of the ARCH-LM test is that there is no ARCH up to lag order *p* in the residuals. The test was completed by collecting the residuals from a chosen model and performing regression between squared residual and lagged-squared residuals. The test statistic of ARCH-LM is calculated by $T \cdot R^2$. The statistic is distributed with a Chi-square, χ^2_p, where *T* is the number of observations, R^2 is the coefficient of determination from the auxiliary regression, and *p* is the number of variables in the right-hand side of the auxiliary equation.

the estimated values of the fractional integration parameter, d, does not change significantly. This suggests that although long memory parameter value is sensitive to the specification of temporal effects, there is no change in the main implication of fractional integration.

The estimates of the GARCH(1,1) and FIGARCH(1,d,1) models for wheat under the normality assumption are displayed in the second and fourth column, respectively, in Table 3. The Q statistics show that the simple model, GARCH(1,1), performs a good job in modeling dependence in the volatility of wheat cash returns. The FIGARCH(1,d,1) performs better only for $Q^2(20)$. The fractional differencing estimate is 0.277, which represents the 0.277th order of integration. The t-statistic of the fractional differencing parameter indicates that the long memory parameter is statistically significant at the 5 percent level. However, the usual Wald type t-test may not provide a correct test for the FIGARCH(1,d,1) over the GARCH(1,1) because the Wald type test is not invariant in nonlinear models. Therefore, a likelihood ratio (LR) test would be better for the test of FIGARCH vs. GARCH. When compared to that of the GARCH(1,1) model, the log-likelihood and AIC values of the FIGARCH(1,d,1) model do not suggest that the FIGARCH(1,d,1) model performs distinctly better than the GARCH(1,1) model.

For corn return series, the Q-statistics of squared residuals of the FIGARCH(1,d,1) are smaller than those of the GARCH(1,1). This implies that the FIGARCH(1,d,1) is better for capturing dependence in the volatility. The log-likelihood and AIC statistics support this and the EGARCH test also show that the FIGARCH(1,d,1) model is more effective than the GARCH(1,1) in reducing misspecification of the conditional variance equation caused by different sign and size of the stochastic shocks. The fractional differencing parameter for corn is 0.127. For soybeans, a convergence could not be achieved for the FIGARCH(1,d,1) model with the normality assumption. This leads us to compare the GARCH(1,1) model with the FIGARCH(1,d,0) model.

A preferred long memory conditional volatility model by Baillie *at al.* (1996) and Tse (1998) is FIGARCH(1,d,0), which is nested on the FIGARCH (1,d,1) model. We further estimated the FIGARCH(1,d,0) model in order to compare the performances of the two competing long memory conditional volatility models. Estimates and diagnostic statistics of the FIGARCH(1,d,0) model are displayed in the sixth column in Tables 3 through 5. For the wheat and corn series, the two competing models have similar performance in capturing dependence. AIC statistic suggests that the FIGARCH(1,*d*,0) model performs slightly better than the FIGARCH(1,*d*,1) model for wheat, but not for corn. For soybeans, when compared with the GARCH(1,1) model, the FIGARCH (1,*d*,0) model performs better in describing the dependence in the variance in terms of the Q-statistics of squared residuals.

5. Relaxing the Normality Assumption

It has been hitherto assumed that the returns series follow the normal distribution. However, the results of the normality tests, presented in Table 2, indicate that the return series have higher kurtosis than that of the normal distribution. Hereafter, we therefore relax the normality assumption and use the student-*t* distribution to compare performance of the models with those under the normality assumption.

Commodity and financial time series have been traditionally assumed to follow the normal distribution with finite mean and variance. The Gaussian assumption is easy to apply to economic analyses, and many of its properties have been revealed to economists so that the distributional hypothesis has been embraced in economic and financial analyses, despite the fact that empirical evidences show distinct anomalies from the normal distribution. Empirical studies have found that economic and financial data show clear abnormal behaviors. Note that the normality assumption is to a certain extent justified by the fact that the QMLE is found to behave quite well, even when non-normal errors are observed. However, the degree of inefficiency of the estimator increases with the degree of departure from normality (Engle and Gonzlez-Rivera, 1991). If returns display fat-tails behaviors, therefore, it is desirable to use the student-*t* distribution to capture the higher observed kurtosis. Bollerslev (1987) and Hsieh (1989) show that when series have higher kurtosis, the student-*t* distribution performs better than other competing distributional models.

If we write the log-likelihood of the standard normal distribution when the conditional mean equation is expressed as in Equation (1), it is

$$(8) \qquad L_{\text{norm}} = -0.5\sum_{i=1}^{T}[\ln(2\pi\sigma_t^2) + (\varepsilon_t^2/\sigma_t^2)],$$

where T denotes the number of observations. In the case of the student-*t* distribution, the log-likelihood becomes

$$(9)\ L_{\text{Stud}} = T[\ln\Gamma(\frac{\nu+1}{2}) - \ln\Gamma(\frac{\nu}{2}) - 0.5\ln(\nu\pi - 2\pi)] - 0.5\sum_{i=1}^{T}[\ln(\sigma_t^2) + (\nu+1)\ln(1+\frac{\varepsilon_t^2}{\sigma_t^2(\nu-2)})],$$

where $\Gamma(\cdot)$ is the gamma function, and denotes the degrees of freedom ($2 < \nu < \infty$).

The estimates of the volatility models under the student-*t* distribution are displayed in the third, the fifth, and the last column, respectively, in Tables 3 through 5. The *Student DF* denotes a tail coefficient when we use the student-*t* density to capture the fat-tails behaviors. A higher *t*-value for the coefficient indicates a fatter-tails behavior of the volatility than that of the normal distribution. The results of wheat and corn prices show that fat-tails are significant and that the conditional volatility models perform better under the student-*t* distribution than those under the normal distribution, based on Q-statistics for dependency and AIC values. For soybeans prices, the results do not clearly show a better performance of the processes under the student-*t* distribution. Rather, they suggest that in the case of soybeans, skewness also needs to be captured in addition to excess kurtosis. This suggests that a *skewed Student-t density* is a proper distributional model, as proposed by Fernndez and Steel (1998), to account for both the asymmetric and fat-tails behaviors at once.

Overall results suggest that, among the competing six different volatility models, the FIGARCH(1,*d*,0) under the student-t density does the best job in modeling volatility of wheat and corn cash price returns, and the FIGARCH (1,*d*,0) under the normality density is the most appropriate for the case of soybeans. The differences are typically small and statistically insignificant between the Q-statistics, which still supports for using FIGARCH type models for the volatility. Purely from the perspective of describing the volatility of grain prices, the FIGARCH type models appear better than the GARCH model. These suggest that fractional integration parameter is required in modeling grain price volatility.

6. Summary and Conclusion

The objective of this study is to suggest a suitable volatility model for the international grain market. There is yet little consensus on the standard for modeling grain price volatility. The contribution of this paper may be placed on demonstrating the importance of modeling both long memory and temporal effects of the volatility processes of the grain prices in one frame and on finding an appropriate long memory stochastic volatility model for the price series. We applied six different volatility models to cash price series of U.S. wheat, corn, and soybeans, which are of most important commodities in the world grain market. We compared the performance of the models in capturing dependence of the price volatility, and also emphasized suitability of the student-*t* density intended to account for non-normal, fat-tailed properties of the data. Overall, the empirical results lead to a conclusion that the grain cash price volatilities exhibit long memory and that the memory is adequately modeled by a fractionally integrated process, which is implemented via parametric procedures in FIGARCH models. At the same time, we find the suitability of the FIGARCH type models under the student-*t* distribution and the competitiveness of the parsimonious FIGARCH(1,*d*,0) model. It is, therefore, desirable to use long memory conditional variance models for analysis of the grain price volatility dynamics.

This paper is exposed to a limitation from the data availability. We utilized only the monthly data, but not other higher frequencies. Therefore, an attention should be paid to applying the models to other frequencies of the data and/or other commodities. This paper does not proceed beyond showing the suitability of the long memory volatility models and recommending utilization of them in modeling volatility processes. We leave the next step for future studies for an analaysis of a clearer cut of fractional differencing for the volatility processes of agricultural commodities, a hedging strategy with the long memory perspective of volatility dynamics, or a forecasting evaluation with the long memory dynamics models.

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