REVISITING THE CLASSIFIED MILK PRICING SYSTEM: SEASONAL AND SPATIAL MILK PRICING IN THE U.S.*

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Keywords
Classified milk, Class I price differentials, Federal Milk Marketing Order, spatial transport optimization model, seasonal pricing, spatial pricing

Abstract
A key policy issue deliberated by the U.S. federal milk marketing policy group involves the setting of spatial price differentials for Class I milk. The Class I price differentials were established in 2000 and remain in use today. These differentials reflect transport and other factors that vary across space. Since 2000, there have been changes in some factors, such as fuel price and supply/demand locations. We examined how the differentials match up with the distribution of shadow prices in a spatial transport model and found that consideration of fuel costs and supply/demand location shifts raises the magnitude of the differentials by 115%. We also found that seasonal shifts are also a factor, particularly for Class I milk, but not for manufacturing milk differentials. In particular, the seasonal differences do appear to be of a magnitude which would suggest that Class I differential levels need to be revisited seasonally. Collectively, the results indicate that it may be desirable to revisit the policy determined price differentials.

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I. Introduction

The monthly average milk price in the U.S. has shown extreme variability. This volatility is caused by the difficulty of balancing short term supply and demand for milk, which is produced daily and is perishable (Manchester and Blayney 2001; Shields 2009). To stabilize the milk price, the federal government developed Federal Milk Marketing Orders (FMMOs), as authorized by the Agricultural Marketing Agreement Act of 1937. The FMMOs system is designed to provide both price support and market stability by establishing minimum prices that handlers are to pay. Accordingly, FMMOs are designed to ensure that a sufficient quantity of Grade A milk is supplied at a stable price.

The FMMO agreement affects milk pricing through a classified price support system and revenue pooling. Under classified pricing, milk is differentiated according to the milk usage product class. Milk used for products are categorized into four classes under clauses 8(d) and 9(r) of the Dairy Industry Act S.N.S.2000. Generally speaking, Class I milk is used for packaged fluid milk products. Class II milk is used in soft manufactured products such as yogurt and ice-cream. Class III milk is used in hard manufactured products such as cheese. Class IV milk is used in any product not included in the other classes such as butter and powder. Under the system, prices paid by handlers for milk used in Class II, III, and IV are based on monthly wholesale market prices for products belonging to each class, as reported by the Agricultural Marketing Service (AMS). These class prices are identical throughout all locations in the U.S. market. On the other hand, the price for Class I milk varies by location, as it involves a predetermined and fixed Class I price differential that varies geographically.

The current Class I differential varies across the U.S. in the range of $1.60-6.00 per hundredweight (cwt). The minimum price that must be paid by fluid milk handlers to producers in the lowest regions is specified as the higher of the milk prices for Class III or Class IV plus the differential, which is $1.60 per cwt. The main reason for the addition of the differential is to compensate dairy farmers for the additional costs of producing Grade A milk and then getting it to market.

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1 Revenue pooling system causes dairy farmers to be paid a weighted average price for all uses of milk in a particular marketing order. The system gives all dairy farmers in a certain marketing order area the same price, which balances market power between them and milk handlers.
Fluid processors in the highest regions must pay at minimum prices of $4.40 per cwt more to receive Grade A milk than those in the lowest regions. The spatially differentiated prices are intended to allow deficit areas to attract Grade A milk from surplus areas to satisfy the demand for fluid milk and to compensate producers for transportation costs, which helps encourage economic efficiency and orderly marketing in regulated markets. A key issue under deliberation by the FMMOs policy group involves the setting of spatial price differentials for Class I milk. The question is how different the milk values should be.

There is the possibility that the Class I price differentials are in need of revision. The Class I price differentials currently being used were largely established in January 2000. Subsequently, there have been small adjustments of differentials made in May 2008, only in the Appalachian (FO5), Florida (FO6) and Southeast (FO7) Marketing Order areas. However, there have been significant changes in the local supply, local demand and transportation costs, which are potentially key factors for determining the spatial milk values. Accordingly, our study aims to estimate the Class I price differentials reflective of current dairy economy. Additionally, this study also offers estimates of pricing surfaces of other classified milk. Secondly, we investigate the impact of altered transportation costs and the supply-demand factor on the shift in price differentials, by analyzing each factor. Third, we evaluate the impact of the seasonal variations in milk supply-demand on spatial milk values.

II. Background and Literature Review

1. Classified pricing system

The concept of Class I price differentials was initially introduced in 1960 (French and Kehrberg). Christ (1980) compared the hauling cost to move Grade A milk to the Class I price differential structure. He concluded that there was a need to increase Class I price differentials to promote the regional movement of milk. Subsequently, many researchers analyzed the impact of Class I price differentials using spatial programming models such as the Dairy Market Policy Simulator (DAMPS) by Novakovic et al. (1979), the Interregional Competition Model (IRCM) by Cox and Jesse (1995), or other self-developed models (Ahn and
Sumner 2009; Yavuz et al. 1996). These models were used to address a variety of economic issues such as market organization and the potential for improving efficiency, the optimal plant size, numbers, and location, as well as transportation arrangements. A representative example is the study by Pratt et al. (1998), who estimated Class I price differentials using the U.S. Dairy Sector Simulator Model (USDSS). The USDSS, however, did not fully reflect the actual situation due to a mismatch between the real locations of processing plants, which were not optimal, and the simulated optimal points: this was a problem since the model was to determine where to locate the plants and how much dairy product to process at each location (Pratt et al. 1997). The vast majority of the current Class I price differentials was established based on the results from USDSS. However, there have been no studies that specifically assessed the adequacy of the current differential structure after 2000, despite the significant changes in the spatial dispersion of milk suppliers and dairy product demand and the rise in transportation costs.

2. Changes in Factors

Transportation costs have risen substantially since 2000 and this increase should be reflected in an increase in the price differentials. There are many factors that determine transportation rates, but the fuel cost (mainly diesel price) is a leading factor and this has recently increased greatly, more than doubling since 2000.

FIGURE 1. Average U.S. diesel prices from 1994 to 2012 ($/gallon)

![Average U.S. diesel prices from 1994 to 2012](chart)

Source: U.S. Energy Information Administration.

Furthermore, there have been geographic shifts in the location of the milk supply. Milk production is moving to the west (Blayney 2002) because the average costs of milk production are lower in the west for a variety of organizational and climatic reasons. The first panel of Table 1 shows the top five states and the lowest
five states in terms of the variation of the supply share between 2000 and 2012. Idaho experienced the largest increase followed by California, Texas, and Michigan. These four states produced 30.4% of U.S. milk in 2000 and 36.9% in 2012. Meanwhile, the production share of Pennsylvania decreased from 6.7% to 5.2%. The standard deviation of percent change from 2000 to 2012 in the 48 milk producing states is 0.63%, which indicates that regional milk supply has experienced a striking change during the period.

The regional distribution of the demand for dairy products has also changed since 2000. The second panel of Table 1 represents the percent changes in the demand share as a function of the population from 2000 to 2012. Texas experienced the largest increase in demand share from 7.47% to 8.37%, followed by Florida, Arizona, Georgia, and North Carolina. New York experienced the largest decrease in the population during the period, from 6.80% to 6.29%, followed by Michigan, Ohio, and Illinois. Pratt et al. (1998) estimated the impact that the spatial shifts in population will have on the Class I price differentials and forecasted the expected differentials with USDSS, but they did not consider the impact of spatial supply shifts on locational milk values. To my knowledge, there has been no research on the effects of fuel prices on Class I price differentials.

Table 1. The supply & demand shares of 48 states (2000 - 2012)

<table>
<thead>
<tr>
<th>Rank</th>
<th>State</th>
<th>Share of milk production</th>
<th>Share of the population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ID</td>
<td>4.30%</td>
<td>6.77%</td>
</tr>
<tr>
<td>2</td>
<td>CA</td>
<td>19.22%</td>
<td>20.88%</td>
</tr>
<tr>
<td>3</td>
<td>TX</td>
<td>3.40%</td>
<td>4.79%</td>
</tr>
<tr>
<td>4</td>
<td>MI</td>
<td>3.35%</td>
<td>4.44%</td>
</tr>
<tr>
<td>5</td>
<td>NM</td>
<td>3.12%</td>
<td>4.07%</td>
</tr>
<tr>
<td>44</td>
<td>KY</td>
<td>1.01%</td>
<td>0.56%</td>
</tr>
<tr>
<td>45</td>
<td>NY</td>
<td>7.13%</td>
<td>6.59%</td>
</tr>
<tr>
<td>46</td>
<td>MO</td>
<td>1.35%</td>
<td>0.70%</td>
</tr>
<tr>
<td>47</td>
<td>MN</td>
<td>5.66%</td>
<td>4.53%</td>
</tr>
<tr>
<td>48</td>
<td>PA</td>
<td>6.65%</td>
<td>5.27%</td>
</tr>
</tbody>
</table>

Source: USDA-ERS (milk production) and U.S. Census Bureau (population).

3. Seasonal variation of supply and demand

There are seasonal variations in the raw milk supply from individual operations as well as throughout the industry as a whole, due to breeding patterns and weather
conditions, especially excessive heat and humidity (Hahn 1999). Figure 2 shows the monthly variations in daily milk yield compared to the average for 2012 in the U.S. and in 4 selected states. The total U.S. milk production increases from January and peaks in the spring and early summer. It then gradually decreases from May and falls to the lowest yields in September and October. In addition, the raw milk supply also shows regionally varying patterns due to climate differences across the county. Florida shows a larger fluctuation while Wisconsin produces relative constant quantities of milk throughout the year.

FIGURE 2. Percentages of monthly variation compared to annual average milk yield per cow, based on data from 2012 (U.S. total and 4 selected states)

Source: USDA/AMS/Dairy Program.

The demand for dairy products also exhibits seasonality. Figure 3 shows the monthly variation for 4 selected dairy products from the 4 classes; fluid milk represents Class I and we selected ice cream for representing Class II, Italian cheese for Class III, and butter for Class IV. Fluid milk consumption is relatively higher in months when school is in season while ice cream consumption is highest in the summer and lowest in the winter, reflecting the climate conditions. Butter consumption fluctuates while Italian cheese is consumed relatively constantly. Collectively, this seasonality in the supply and demand may well have an influence on monthly differentials for classified milk across the U.S., and in turn could be reflected in the FMMO pricing surface. Testuri, Kilmer, and Spreen (2001) provided insight into the seasonality of Class I price differentials in the Southeastern area of the U.S. using a minimum cost network flow model. However, the study has not been extended across the U.S.
Revisiting the Classified Milk Pricing System: Seasonal and Spatial Milk Pricing in the U.S.

III. Model and Economic Theory

1. MilkOrdII model

For this study, we developed a linear programming model, MilkOrdII, which represents the U.S. dairy sector and integrates and extends the features of previous models. It is formulated as a spatial transport model that incorporates economic activity at farms, dairy product plants, and consumer markets including export and stock storage levels. Many previous models have been price endogenous models (Enke 1951; Samuelson 1952) in which supply and demand curves are applied to determine the equilibrium quantity and price. Solutions to the models are obtained by maximizing the consumer and producer surplus, given that market behavior is competitive. Since the main purpose is to estimate the milk movements, processing, and price differentials fully reflecting the current dairy economy, however, MilkOrdII uses a fixed production and consumption model (Stollsteimer 1963; Enke 1951; Samuelson 1952) in which supply and demand curves are applied to determine the equilibrium quantity and price. Solutions to the models are obtained by maximizing the consumer and producer surplus, given that market behavior is competitive. Since the main purpose is to estimate the milk movements, processing, and price differentials fully reflecting the current dairy economy, however, MilkOrdII uses a fixed production and consumption model (Stollsteimer 1963; Enke 1951; Samuelson 1952).

MilkOrdII expands on the model as adapted from McCarl’s earlier work (McCarl, Schwartz, and Siebert, 1996) that created the first version of MilkOrd which integrated features from the DAMPS model by Novakovic et al. (1979) and the dairy processing model of Baker, Dixit, and McCarl (1981). The MilkOrdII contains 163,927 constraints and 8,768,678 variables. It was solved using GAMS and took approximately two hours of CPU time to obtain an optimal solution without the use of an advanced basis.

FIGURE 3. Percentages of monthly variation compared to the annual averages of consumption per day for 4 selected dairy products, based on data from 2012

Source: USDA/AMS/Diary Program.
Ladd and Halvorson 1970) of interregional trade with fixed supply, consumption, and plant capacity. It assumes that the seasonal supply of raw milk in pounds and the demand for dairy products are exogenous over the simulation time and the commodity price adjusts to meet the equilibrium conditions.

The model contains data for 12 months in a year, reflecting seasonality. Month to month storage allows carryover of available dairy private stocks. If milk supplies are large relative to demand, then the supply of milk that is not needed for perishable products will increasingly be diverted to the manufacture of storable products. Once the products are made, they can be placed in private storage. When milk supplies are tight relative to demand, then storable products are released to the commercial market from private storage. A remarkable feature of MilkOrdII is that it models products and their composition in a different manner. Previous models used milk components such as fat and non-fat solids to account for the balances among raw milk supply, inter-plant transfers of dairy products, and final product consumption (Ahn and Sumner 2009; Testuri, Kilmer and Spreen 2001; Yavuz et al. 1996; Cox and Jesse 1995; Pratt et al. 1998; Novakovic et al. 1980). MilkOrdII incorporates the unit conversions for each process obtained from the processing model (Baker, Dixit and McCarl 1981). It can determine the amount of each dairy product based on fixed input-output volume ratios of raw ingredients to final products at the plant level (i.e. a given amount of milk yields a fixed proportional amount of low fat milk and cream). The only exception is for ice cream mixes and cottage cheese dressings, where there is a blending problem when milk components are mixed with a maximum of whey content.

MilkOrdII breaks down the continental U.S. into 303 regions, corresponding to the crop reporting districts defined by the National Agricultural Statistical Service (NASS). Raw milk is represented by grade; it is divided into Grade A, Grade B, and unregulated milk at the farm level, since each type of milk is intended for a different usage. Raw milk at the plant level is classified in a range from Class I to Class IV according to the type of plant the milk is destined for. The model includes 9 different kinds of plants: Class I type plants (fluid plants), Class II type plants (yogurt, ice cream, sour cream, and cottage cheese plants), Class III type plants (Italian cheese, cheddar cheese plants), and Class IV type plants (butter and powder plants). There are 15 representative processes at the plant-level. The model represents the production of raw milk into a total of 25 dairy products; 23 of these are fixed proportion blends of intermediate or final products, and 24 are mixed products. In terms of intermediate products, some dairy
products produced in plants are not directly delivered to consumer markets but rather are transferred to another plant as ingredients for other products. For example, excess cream from a fluid plant can be transferred to a sour cream plant and used to make sour cream. However, cream is also a kind of final product, since it is distributed to consumer markets to satisfy the cream demand.

\[ (1) \quad \text{MIN} \quad Z = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{c=1}^{C} \sum_{t=1}^{T} \overline{AC}_{i,j,t} \ast (XGA_{i,j,c} + XGB_{i,j,c} + XSP_{i,j,c} + XUF_{i,j,t}) + \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{p=1}^{P} \sum_{t=1}^{T} \overline{DC}_{i,j,p,t} \ast (XPI_{i,j,p,t} + XPD_{i,j,p,t} + XSR_{i,j,p,t} + XSA_{i,j,p,t}) + \sum_{i=1}^{I} \sum_{r=1}^{R} \sum_{t=1}^{T} \overline{PC}_{i,r,t} \ast QDP_{i,r,t} + \sum_{i=1}^{I} \sum_{p=1}^{P} \sum_{t=1}^{T} \overline{M} \ast (AGA_{i,t} + ADP_{i,p,t}) - \sum_{i=1}^{I} \sum_{p=1}^{P} \overline{TS}_{i,p} \ast QSP_{i,p,t} \]

The objective function is to minimize total costs incurred within the U.S. dairy industry during one year, subtracting the revenues from the terminal values of stocks at the ending of final month. The first part in the equation (1) is the assembly cost to ship Grade A \((XGA_{i,j,c})\), Grade B \((XGB_{i,j,c})\), unregulated \((XUF_{i,j,t})\), and supplying milk \((XSP_{i,j,c})\). The assembly rate per unit \((\overline{AC}_{i,j,t})\), is identical regardless of the type of raw milk or type of classified milk. The second is the transport cost of dairy products including the inter-transfer cost of intermediate products \((XPI_{i,j,p,t})\), the distribution cost of final products \((XPD_{i,j,p,t})\), and the shipping cost of storable products \((XSR_{i,j,p,t} \text{ and } XSA_{i,j,p,t})\). The third is the production cost to make dairy products \((QDP_{i,r,t})\). The fourth refers to big penalties \((M)\) related to positive artificial variables \((AGA_{i,t} \text{ and } ADP_{i,p,t})\). Additionally it includes terminal values \((\overline{TS}_{i,p})\) for the amount of stocks in the final month.

3 These include fluid milk, skim milk, yogurt, cream, ice cream, sour cream, cottage cheese, Italian cheese, cheddar cheese, condensed skim milk, condensed whole milk, butter, non-fat-dry, powder, whey butter, butter milk, cottage cheese whey, mozzarella cheese whey, cheddar cheese whey, dry butter milk, dry cottage cheese whey, dry mozzarella cheese whey, and dry cheddar cheese whey.

4 Ice cream mixes are used to produce ice cream, and cottage cheese dressing is utilized to make cottage cheese. They are made by blending several products with raw milk.
(\(Q_{SP_f,p,t}\)) to ensure that the model activity is reasonable up until the final month. To obtain the stock values, we ran the MilkOrdII model with the object of minimizing total costs, and observed the shadow prices in the balance of stocked products. With the shadow prices, we ran the model again to ensure that the model activity is reasonable up until the final month.

The set of constraints can be broadly classified into seven groups. First, there are raw milk supply/demand balance constraints at the level of farmers and plants. Second, there are constraints that balance intermediate/final products and mixed products at the processing plants. Third, there are blending constraints on ice cream mixes and cottage cheese dressings. Fourth, there are final product demand constraints at the level of consumers. Fifth, there are stock balance constraints requiring that the stock from the previous month, with the addition of added stock and subtraction of released stock in the current month must be equal to the amount of stock in current month. Sixth, there are plant capacity constraints restricting the maximum amount of manufacturing. Lastly, several sets of constraints are added in order to implement the model on an even greater level of ‘real-world’ structure.

Figure 4 represents the movement of raw milk and milk products from farms to plants in the model. Since Grade A milk can be used for all classified dairy products, it can be used as Class I or downgraded to a lower class level. Grade B milk is only used for manufactured products (Class III and Class IV). Unregulated milk can be used for fluid milk as well as manufacturing milk. Some milk is shipped to a supply plant which in turn reships the milk to other processing locations. While engaged primarily in manufacturing, ‘supply plants’ help ensure that there is an adequate supply of milk for fluid by carrying fluid milk reserves. Raw milk and dairy products can be shipped from departure to arrival regions subject to maximum distance limits, which restrict how far they can be transported.

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5 A mathematical formulation of the model as well as the detailed description of constraints are shown in the Appendix.

6 When milk is needed for fluid purposes, supply plants are required to ship milk to fluid processors rather than use the milk to make manufactured dairy products. Supply plants provide a “balancing” service by manufacturing milk that is not needed for fluid purposes on days when bottling plants are not operating.
2. Relative shadow prices as pricing differentials

The primal solution from MilkOrdII gives the least cost spatial pattern for milk movement and processing along with dairy product movement, and stock accumulations as well as release flows, given the fixed supply, demand, and maximum capacity at disaggregated regions during a 12-month time period. More importantly, the model’s associated dual solution provides the marginal values of milk at each location. That is, the marginal values of milk are provided from the milk demand balance constraint at the plant-level as shown below.

\[
\sum_{l=1}^{L} \sum_{r=1}^{R} QRI_{c,l,r} \cdot QDP_{i,l,r,t} + \sum_{m=1}^{M} QRM_{i,c,m,t} \leq QRP_{i,c,t} \forall i \in I, \forall t \in T, \forall c \in C
\]

The constraint means that the milk supply by Class, \( QRP_{i,c,t} \), is balanced with milk use to make dairy products, \( \sum_{l=1}^{L} \sum_{r=1}^{R} QRI_{c,l,r} \cdot QDP_{i,l,r,t} \), or to blend into mixed products, \( \sum_{m=1}^{M} QRM_{i,c,m,t} \). Here, \( QRI_{c,l,r} \) is the amount of \( c^{th} \) classified milk used for a unit of \( i^{th} \) process at \( l^{th} \) plant and \( QDP_{i,l,r,t} \) denotes the amount of \( i^{th} \) process at \( l^{th} \) plant in \( r^{th} \) region in \( t^{th} \) month. Its associated dual solution \( \lambda_{i,c,t} \) is represented as:
These values, the shadow prices, give the marginal value of more milk at a location in the optimal solution. Since the constraints (2) are constructed for classified milk in each region, the shadow price of each classified milk can be obtained as (3). The shadow prices at a fluid processor can be interpreted as follows: if a handler at a location obtained one more unit of milk, then the entire cost involved with distribution of raw milk and dairy products will be reduced by the amount of the shadow price. This concept is consistent with economic theory on how prices are determined in a competitive market (Samuelson 1952).

However, the derived value from the model does not yield the absolute value or Class I differentials since these reflect only the ‘transportation’ derived component of locational differentials in terms of relative differences. Other components, such as milk production cost and/or marketing margins are not included in the model. Nonetheless, the relative shadow price between different regions can be used as a measure of relative Class I price differentials across the regions under the assumption that there is homogeneity in the processing costs and milk/product composition across the U.S.\(^7\) Therefore, the simulated shadow prices are used to provide information regarding price differentials among geographic locations. More specifically, the differences of the shadow prices imputed from fluid milk demand constraints between two regions are equivalent to the differences of Class I price differentials between them.

To obtain the locational differentials \((A_{i,c=1,t})\), the Class I milk shadow prices \((\lambda_{i,c=1,t})\) that we derived from MilkOrdII are adjusted in this manner:

\[
(4) \quad A_{i,c=1,t} = \lambda_{i,c=1,t} -\text{MIN}(i, \lambda_{i,c=1,t}) \quad \forall i \in I, \forall t \in T, \forall c \in C
\]

\(^7\) Since it is impossible to collect NASS district-level data for processing costs and milk production costs, the heterogeneity of those factors are not considered in the model. If we are able to use the production costs data, we should include the regional production costs, which is the multiplication of raw milk supply and production costs in each NASS district, into the objective function. Also, if we gather the processing costs data, we should take account of it by identifying the heterogeneity of regional processing costs. This means that \(\overline{PC}_{t,r,c}\) will be changed to \(\overline{PC}_{t,r,c,t}\)(the processing cost per unit of \(i^{th}\) process at \(r^{th}\) plant in \(c^{th}\) region in \(t^{th}\) month).
That is, the minimum Class I shadow price for each month is subtracted from all shadow prices yielding a base value of zero, and other values ranging up to the highest differential. These values, interpreted literally, indicate the relative change in the optimal objective value resulting from one unit of change in the availability of raw milk at the location in comparison to other locations or equivalently the optimal relative valuation of milk delivered to a location. As noted above, these differentials reflect only the component of spatial differentials derived from ‘transportation,’ since other differential components are not included in the model.

3. Data and Method

3.1 Production, processing, and consumption data

To determine mathematically consistent spatial values for milk across the country, we used the current data for the year 2012. Raw milk production data are developed by the USDA/AMS/Dairy program, applying the three grade categories at the geographic level of the NASS district. Since the seasonal variation of the milk supply is varies across the U.S. we did not use the U.S. total variation but instead applied the 23 selected states variations to the seasonality of each 303 regions. In the case of regions in non-selected states, we used an monthly average variations from neighboring states for which data was available.

Since there are no available surveys or published consumption data at the level of states or NASS districts, we used per capita consumption for each product and the population for each region to calculate the consumption amount for each region. This approach is based on an assumption of constant per capita consumption across the U.S. To reflect consumption seasonality, we calculated the U.S. monthly consumption index for each dairy product, based on the published data from USDA-AMS. For some products, we were unable to obtain consumption data, and perforce used the monthly U.S. production data available from USDA-ERS as a proxy for the consumption data, based on the assumption that the monthly production of dairy products roughly matches its consumption and that long term storage is not permitted. The unit conversion rate at the processing sector was assembled by the USDA/AMS/Dairy program. The program also collected plant capacity data on the basis of how the milk was used at the geographic level of each district. The capacity is assumed to be invariant during the year.
3.2 Transportation costs

Since one of main priorities is to study the effect of fuel prices on the pricing surface, we estimated the impact of the diesel price on unitary transportation costs using the following equation.

\[(5) \quad \text{Transportation cost} = \text{distance} \times (\beta + \gamma \times \text{Diesel Price}) + \alpha\]

The unitary transportation costs between two regions consist of variable costs linearly increasing with distance and fixed costs independent of distance. Fuel costs, driver labor costs, and vehicle maintenance costs are assumed to be a function of distance, and we divided them into fuel costs \((\gamma \times \text{Diesel Price})\) and other factors \((\beta)\). Fixed costs \((\alpha)\) independent of distance include rolling stock, handling costs, milk testing costs, truck replacement costs, etc. The California Department of Food and Agriculture surveyed the hauling rate for each path conveying shipment across 13 sub-areas in the California Marketing Order twice a year from 2006 to 2013. We used that dataset in the estimation since it corresponds to the dimension and interests of MilkOrdII model. For the diesel price data, we used the monthly average highway-diesel prices available from the U.S. Energy Information Administration. The panel data set consists of 577 observations with 58 routes over 15 months. From equation (5), we derived the following panel model.

\[(6) \quad \text{Rate}_{i,t} = \alpha + \beta \text{Dist}_{i,t} + \gamma \text{Dist}_{i,t} \times \text{Diesel Price}_t + u_i + e_{i,t}\]

where \(\text{Rate}_{i,t}\) is the average transportation cost per hundredweight for an individual route \(i\) in month \(t\), \(\text{Dist}_{i,t}\) is the average transport distance for an individual route \(i\) in month \(t\), and \(\text{Dist}_{i,t} \times \text{Diesel Price}_t\) is an interaction term with distance and diesel price in dollars per gallon. Since each route has different roads and other conditions, the unknown route-specific term \(u_i\) is included in the equation, and \(e_{i,t}\) is the idiosyncratic error term. In estimation, we employed a random effects approach\(^8\), and found that every estimate is statistically significant at the 1% level. In turn, the transportation cost per full (48,000 lb) load is determined to be

\(^8\) To decide on the panel estimation method, we run the Hausman test and Breusch-Pagan Lagrange Multiplier tests, and conclude that the random effects approach is reasonable to use in estimating the model. Also we find that the test for homoscedasticity is not passed, and thus use the STATA option ‘robust’ to control for heteroscedasticity.
These results indicate that the fixed cost per truck is $134 per month, and the variable cost of non-diesel inputs is $1.6 per mile. If the diesel price per gallon increases by $1, then the transportation cost of a full road will increase by $0.325 per mile. Unitary transportation cost for each path is calculated with the estimated equation (7) given the distance between two regions and the diesel price.

Since each dairy product requires three different types of transportation, the distribution costs will vary depending on the transport type. The distance data for each path were derived from MPMileCharter with Microsoft MapPoint. Since the populated area, dairy farm area, and plants area are not consistent in each NASS district, the distance for each path is derived for three types of shipments, namely, (1) the raw milk assembly distance between the main dairy farm area of the shipping NASS district and the primary plants area of the receiving NASS district, (2) the inter-transfer shipments distance between the plant-area of the shipping NASS district and the plant-area of the receiving NASS district, and (3) the final product distribution distance between the plant-area of the shipping NASS district and the most populated area of the receiving NASS district.

3.3 Simulation to evaluate the impact of each factor

We conducted separate simulations to discern the impact of each of the three factors. First, we simulated a case with only changing diesel prices where we converted them to 2000 levels in the equation (7). Secondly, we simulated a case by changing only the pattern of demand. This was done by maintaining the total consumption at 2012 levels but rearranging the demand shares among the NASS districts based on the population shares in 2000. In this manner, we were able to isolate the impacts of the spatial shifts in population over time. Thirdly, we simulated a case by changing only the pattern of the raw milk supply back to the 2000 distribution but again maintaining the milk supply volume at the 2012 level. The fourth case was to change both the supply and demand patterns from 2012 to 2000, and the last case was to change all three factors to the 2000 level.
IV. Results

1. Estimated spatial milk values across the U.S.

Since the purpose of our study is to create ideal spatial distribution of Class I differentials and compare it with the existing one, we normalized the current differentials so that the minimum value is zero, and the range is from $0 to $4.4/cwt. Figure 5 depicts a contour map of the normalized actual Class I differentials based on the 303 MilkOrdII regions. The actual differentials generally increase in a ‘regular’ fashion with distance to the east and south of the Upper Midwest, but there is little regularity of increasing differentials to the west. The contour map on the left in Figure 6 represents the estimated Class I pricing surface derived by MilkOrdII. Similar to the current Class I differential structure, the values increase from low values in the northwest to high values in the southeast, showing that MilkOrdII performs well in replicating the general pattern of the Class I differential structure. However, the model-derived Class I value surface has much larger differentials than under the current surface. The lowest valued area is northern New York, and other relatively low valued areas include Minnesota, Wisconsin, Iowa, and North Dakota. Southern Florida, at $9.48/cwt, is the highest valued area.

FIGURE 5. Pricing surface of actual Class I differential structures ($/cwt)
FIGURE 6. Pricing surface of MilkOrdII-generated Class I & Class II milk in 2012

Table 2 shows the range, weighted average, and standard deviation of spatial differentials for classified milk based on 2012 data. The range of simulated differentials is about $5/cwt greater than that of actual differentials. The weighted average differential (weighted by the Class I sales estimates) is $4.03/cwt and is $1.39/cwt greater than weighted average of the current differentials. The standard deviation of $1.93/cwt indicates that the disparity in the MilkOrdII generated pricing surface is much larger than under the current surface. The results imply that the current Class I price differentials are not fully reflective of increased fuel prices nor of changes in local demand/supply conditions.

MilkOrdII also generates manufacturing milk spatial differentials for each type of classified plants; the map for Class II is presented on the right side of Figure 6 and maps for Class III and Class IV differentials are presented in Figure 7. All three pricing surfaces show similar patterns increasing gradually and somewhat uniformly from the west to the southeast. The range of Class II price differentials is $8.32/cwt. and the standard deviation of those is $1.37/cwt, which indicates that Class II milk values are significantly different across geographically separate locations, as in the case of Class I milk values. On the other hand, the ranges of estimated differentials for Class III ($3.05/cwt) and Class IV milk ($4.03/cwt) are much smaller than those of Class I and Class II. Furthermore, the weighted average differentials ($0.50/cwt for Class III and $0.57/cwt for Class IV) and the standard deviation of differentials ($0.47/cwt for Class III and $0.72/cwt
for Class IV) are less than $1/cwt, which indicates that Class III and Class IV milk surfaces are fairly uniform across the U.S. The results correspond somewhat to the current pricing system, which uses identical prices for manufacturing milk across the U.S.

### TABLE 2. Actual and MilkOrdII-generated differentials for classified milk in 2012 ($/cwt)

<table>
<thead>
<tr>
<th></th>
<th>Class I price differentials</th>
<th>Manufacturing milk differentials</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual(a) Simulated(b) (a)-(b)</td>
<td>Class II</td>
</tr>
<tr>
<td>minimum</td>
<td>0.00 0.00 -0.70</td>
<td>0.00</td>
</tr>
<tr>
<td>maximum</td>
<td>4.40 9.48 5.08</td>
<td>8.32</td>
</tr>
<tr>
<td>weighted average</td>
<td>2.64 4.03 1.39</td>
<td>1.78</td>
</tr>
<tr>
<td>standard deviation</td>
<td>0.77 1.93</td>
<td>1.37</td>
</tr>
<tr>
<td>count</td>
<td>303 159 159</td>
<td>134</td>
</tr>
</tbody>
</table>

2. Analyzing the supply, demand, and fuel price effects

The map on the upper left of Figure 8 represents the estimated Class I price differentials from the baseline case of 2012. The map on the right shows the results based on diesel price reverted to 2000 levels. The surfaces are similar in pattern, but the total differential is much smaller with the 2000 diesel price cutting the range to $5.50/cwt, which is 58% of the range under the 2012 prices. The range

Note: Red points indicate the regions with classified type of plants. 86 regions have Class III type plants and 44 regions have Class IV type plants.
of simulated differentials is much more similar in magnitude to the current FMMO differentials; it is only $1.10/cwt greater. We concluded that the fuel price is a key factor in the larger differentials from MilkOrdII and that the ones from FMMO should be adjusted to reflect the increasing fuel prices by applying a formula, since prices will undoubtedly change in the future.

The maps in the center left and right show the simulated differentials under the spatial shift scenarios of demand and supply, respectively. The change in spatial demand patterns does not have major effects on the surface. The differentials derived from supply shifts increase in a ‘regular’ fashion to the west whereas there is no regularity of increasing differentials from the Upper Midwest to the west in the pricing surface from base case. All in all, we found that the supply spatial reallocations have a greater impact on the associated differentials than the demand shifts during the period.

FIGURE 8. Pricing surface of MilkOrdIII-generated Class I milk based on five different scenarios and current structures ($/cwt)
When the model was run under the simultaneous shifts in all three factors, the simulated pricing surface (lower left map in figure 8) became fairly similar to the current structure of Class I differentials (see the map in the lower right of Figure 8). As summarized in Table 3, the range of differentials is $4.86/cwt, which is only $0.46/cwt greater than that of the current differential structure. Also, the standard deviation of spatial differentials is $1.00/cwt, which is only $0.23/cwt greater. It implies that the current differential structure is reflective of the conditions in 2000, and we suggest that these should be updated to reflect the subsequent changes in spatial and fuel costs.

**TABLE 3.** Class I price differentials estimated from five different scenarios ($/cwt)

<table>
<thead>
<tr>
<th></th>
<th>Base*</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>range</td>
<td>4.40</td>
<td>9.48</td>
<td>5.50</td>
<td>9.18</td>
<td>9.46</td>
<td>8.42</td>
</tr>
<tr>
<td>W. AVG</td>
<td>2.64</td>
<td>4.03</td>
<td>2.34</td>
<td>4.61</td>
<td>4.85</td>
<td>5.30</td>
</tr>
<tr>
<td>STDV</td>
<td>0.77</td>
<td>1.93</td>
<td>1.11</td>
<td>2.27</td>
<td>1.78</td>
<td>1.73</td>
</tr>
</tbody>
</table>

*‘S’ implies supply, ‘D’ implies demand, and ‘F’ implies fuel price data.*
3. Impact of supply/demand seasonality on milk values

Figure 9 shows the monthly range of MilkOrdII-generated Class I price differentials throughout 2012. It shows that the largest magnitude of the differentials was $13.86/cwt in October which is almost 90% greater than the smallest magnitude, which was found in June ($7.28/cwt). This is caused by the seasonality of raw milk production and fluid milk consumption, as shown in Figure 2 and Figure 3. It indicates that the months with the largest differentials correspond to the months with the highest demands for fluid milk relative to the raw milk supply. Accordingly, we found that milk seasonality significantly impacts the differentials, which indicates that it might be appropriate to establish seasonally varying differentials.

FIGURE 9. Monthly variations in the range of Class I price differentials ($/cwt)

V. Discussion and Conclusion

Using a spatial milk transport and processing model, this study explored the relative price differentials of classified milk based on data from 2012. We found that the Class I milk values simulated by the model span a total range of $9.48/cwt from the lowest to the highest valued locations, which is much greater than that found in the FMMO Class I prices ($4.40/cwt). We also found a large span in Class II milk prices but a relatively flat surface for manufacturing milk.
We analyzed the reasons for this large price span and found that the differences between the simulated differentials and the FMMO differentials arose largely because of changes in three factors since the time the FMMO differentials were established, mostly in 2000. More specifically, the differences are largely explained by (1) increasing fuel prices, (2) spatial shifts in the location of the supply, and (3) spatial shifts in the location of demand. This indicates that the FMMO differential structure ought to be realigned to reflect these developments. Since the variability of fuel prices is large, the set of Class I differential values might be reconsidered more often, perhaps using a formula that includes fuel prices. In addition, since the readjusted Class I price differential structure will undoubtedly cause changes in the locational valuation in the long-term, the federal milk marketing policy group should continue to examine the trends and magnitudes of change in the regional supply/demand share and realign the Class I differential structure if required.

We also found that seasonality has substantial effects on the monthly estimated Class I price differentials and that it may be desirable to consider establishing Class I differentials on a seasonal basis. The results suggest that there is room to update the Class I differentials to reflect the current dairy economy and the seasonal variation of differentials caused by supply/demand variation, even though the issue of how great the changes in differential values should be and how frequently such changes should be made remains subject to political negotiation and debate.

The current milk pricing system in Korea depends only on milk production costs, which means that it does not reflect changes in factors on the demand side. The system does not allow us to adjust milk prices even though a glut of milk continues to exist in the Korean dairy market. Accordingly, the classified milk pricing system in the U.S. can be a good alternative to the current system for equilibrating the demand and supply, since Class III and Class IV milk prices are determined by a market mechanism. When the classified milk pricing system is considered as an alternative, the Class I price differentials in Korea will need to be analyzed as well.
REFERENCES


Date Submitted: Oct. 10, 2016
Appendix: Mathematical Formulation of MilkOrdII

■ SETS

\( i, j \in I, J; \quad I, J = 303 \text{ regions according to NASS districts} \)

\( i^P \in I^P \subset I; \quad I^P = 6 \text{ regions allowing for supply plants} \)

\( i^S \in I^S \subset I; \quad I^S = 15 \text{ regions with facilities for private stock storages} \)

\( i^E \in I^E \subset I; \quad I^E = 37 \text{ regions exporting dairy products into the world market} \)

\( a \in A; \quad A = 12 \text{ segmented areas; 10 FMMOs, California State Marketing Order, and unregulated} \)

\( a^P \in A^P \subset A; \quad A^P = 3 \text{ FMMO areas allowing supply plants} \)

\( a^F \in A^F \subset A; \quad A^F = 10 \text{ FMMO areas} \)

\( t \in T; \quad T = 12 \text{ months in a year} \)

\( t^B \in T^B \subset T; \quad T^B = \text{January} \)

\( t^E \in T^E \subset T; \quad T^E = \text{December} \)

\( c \in C; \quad C = 4 \text{ differentiated milk according to milk usage product} \)

\( c^M \in C^M \subset C; \quad C^M = 2 \text{ differentiated milk used for manufactured dairy products} \)

\( p \in P; \quad P = 23 \text{ final or intermediate dairy products} \)

\( p^B \in P^B \subset P; \quad P^B = 6 \text{ dairy products used to make mixed products} \)

\( p^W \in P^W \subset P; \quad P^W = 3 \text{ dry whey products} \)

\( p^S \in P^S \subset P; \quad P^S = 4 \text{ dairy products available for private stocks} \)

\( m \in M; \quad M = 2 \text{ mixed products} \)

\( l \in L; \quad L = 9 \text{ different kinds of plants} \)

\( r \in R; \quad R = 15 \text{ different types of production processes at plant} \)

\( ch \in CH; \quad CH = 4 \text{ milk components consisting of products and raw milk} \)

■ PARAMETERS

\( QGA_{i,t}; \quad \text{The amount of Grade A milk supply from } i^{th} \text{ region in } t^{th} \text{ month} \)

\( QGB_{i,t}; \quad \text{The amount of Grade B milk supply from } i^{th} \text{ region in } t^{th} \text{ month} \)

\( QGU_{i,t}; \quad \text{The amount of unregulated milk supply from } i^{th} \text{ region in } t^{th} \text{ month} \)

\( QPD_{i,t,p}; \quad \text{The amount of demand for } p^{th} \text{ product in } i^{th} \text{ region in } t^{th} \text{ month} \)

\( CP_{i,c}; \quad \text{The maximum plant capacity in terms of } c^{th} \text{ classified milk in } i^{th} \text{ region} \)

\( AC_{i,j,t}; \quad \text{The assembly cost per unit of raw milk from } i^{th} \text{ region to } j^{th} \text{ region in } t^{th} \text{ month} \)

\( DC_{i,j,p,t}; \quad \text{The distribution cost per unit of } p^{th} \text{ product from } i^{th} \text{ region to } j^{th} \text{ region in } t^{th} \text{ month} \)

\( QRI_{i,l,r}; \quad \text{The amount of } c^{th} \text{ classified milk used for a unit of } r^{th} \text{ process at } l^{th} \text{ plant} \)

\( QPI_{p,l,r}; \quad \text{The amount of } p^{th} \text{ product used for a unit of } r^{th} \text{ process at } l^{th} \text{ plant} \)
\(\overline{QMI}_{m,t,r}\): The amount of \(m^{th}\) mixed product used for a unit of \(r^{th}\) process at \(t^{th}\) plant

\(\overline{QPO}_{p,t,r}\): The amount of \(p^{th}\) product made from a unit of \(r^{th}\) process at \(t^{th}\) plant

\(\overline{PC}_{l,r,t}\): The production cost per unit of \(r^{th}\) process at \(t^{th}\) plant in \(t^{th}\) month

\(\overline{RCH}_{ch}\): The percentage of \(c_{h}\) th component in raw milk

\(\overline{PCH}_{p, ch}\): The percentage of \(c_{h}\) th component in \(p^{th}\) product

\(\overline{MCH}_{m, ch}\): The percentage of \(c_{h}\) th component in \(m^{th}\) mixed product

\(\overline{MIS}_{g, ch}\): The minimum private stock of \(p^{th}\) product in \(i^{th}\) region

\(\overline{QBS}_{i, p, t, r}^{s}\): The amount of private stock of \(p^{th}\) product in \(i^{th}\) region at the beginning of \(t^{th}\) month.

\(\overline{TS}_{p}\): The terminal values of \(p^{th}\) storable product at the ending of final month

\(\overline{PS}_{a}^{f}\): The minimum percent of Class I milk shipped to supply plants in \(a^{th}\) MMOs

\(\overline{MCU}_{f, c}\): The minimum use percent of \(c^{th}\) classified milk capacity in \(a^{th}\) MMOs

\(\overline{MAW}_{m}\): The maximum percentage of dry whey products used in the \(m^{th}\) mixed product

\(\alpha\): The maximum percent of unregulated milk used for fluid

\(load\): The minimum amount of Grade A milk shipped to fluid plants

\(M\): A big positive number

### DECISION VARIABLES

\(\overline{XGA}_{i, j, c, t}\): The amount of \(c^{th}\) classified Grade A milk shipped from \(i^{th}\) region to \(j^{th}\) region in \(t^{th}\) month

\(\overline{XGB}_{i, j, c, t}\): The amount of \(c^{th}\) classified Grade B milk shipped from \(i^{th}\) region to \(j^{th}\) region in \(t^{th}\) month

\(\overline{XSP}_{i, p, c, t}\): The amount of supplying milk shipped from \(i^{th}\) region to \(j^{th}\) region in \(t^{th}\) month, where \(c \in \{\text{Class I}\}\)

\(\overline{XSP}_{i, j, c, t}\): The amount of supplying milk shipped from \(i^{th}\) region to \(j^{th}\) region in \(t^{th}\) month, where \(c \in \{\text{Class I}\}\)

\(\overline{XUF}_{i, t}\): The amount of unregulated milk used for fluid milk in \(i^{th}\) region in \(t^{th}\) month

\(\overline{SGA}_{i, t}\): The amount of Grade A milk downgraded to \(c^{th}\) classified milk from \(c-1\) th classified milk in \(i^{th}\) region in \(t^{th}\) month, where \(c \in \{\text{Class II, III, IV}\}\)

\(\overline{SGB}_{i, c, t}\): The amount of Grade B milk downgraded to \(c^{th}\) classified milk from \(c-1\) th classified milk in \(i^{th}\) region in \(t^{th}\) month, where \(c \in \{\text{Class IV}\}\)

\(\overline{SNB}_{i, t}\): The amount of unregulated milk converted to manufacturing milk in \(i^{th}\) region in \(t^{th}\) month, first switched to Class III milk

\(\overline{QRP}_{i, t}\): The amount of \(c^{th}\) classified milk which \(i^{th}\) region receives in \(t^{th}\) month

\(\overline{QRM}_{i, c, m, t}\): The amount of \(c^{th}\) classified milk used to make \(m^{th}\) mixed product in \(i^{th}\) region in \(t^{th}\) month, where \(c \in \{\text{Class II}\}\)
Revisiting the Classified Milk Pricing System: Seasonal and Spatial Milk Pricing in the U.S.

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The amount of \( \text{process at } i^{th} \text{ plant in } t^{th} \text{ region in } \) month

The amount of \( \text{final product shipped from } i^{th} \text{ region to } j^{th} \text{ region to satisfy consumer demand in } t^{th} \text{ month} \)

The amount of \( \text{intermediate product shipped from } i^{th} \text{ region to } j^{th} \text{ region to be used for production in } t^{th} \text{ month} \)

The amount of \( \text{product used to make } m^{th} \text{ mixed product in } i^{th} \text{ region in } t^{th} \text{ month} \)

The amount of \( \text{stock product added to } j^{th} \text{ stock region from } i^{th} \text{ region in } t^{th} \text{ month} \)

The amount of \( \text{stock product released from } i^{th} \text{ stock region to } j^{th} \text{ region in } t^{th} \text{ month} \)

The amount of \( \text{mixed product made in } i^{th} \text{ region in } t^{th} \text{ month} \)

The amount of \( \text{product sold with fixed price in } i^{th} \text{ region in } t^{th} \text{ month} \)

The amount of \( \text{stocks stored in } i^{th} \text{ region at the end of } t^{th} \text{ month} \)

The insufficient amount of Grade A milk supply in \( i^{th} \text{ region in } t^{th} \text{ month} \)

The unsatisfied demand of \( \text{final product in } i^{th} \text{ region in } t^{th} \text{ month} \)

The objective function value, i.e. minimized total costs

\[ Z = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{c=1}^{C} \sum_{t=1}^{T} AC_{i,j,t} \left( XGA_{i,j,c,t} + XGB_{i,j,c,t} + XSP_{i,j,c,t} + XUF_{i,j,t} \right) \]

\[ + \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{p=1}^{P} \sum_{t=1}^{T} DC_{i,j,p,t} \left( XPI_{i,j,p,t} + XPD_{i,j,p,t} + XSR_{i,j,p,t} + XSA_{i,j,p,t} \right) \]

\[ + \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{r=1}^{R} \sum_{t=1}^{T} PC_{i,l,r,t} \cdot QDP_{i,l,r,t} \]

\[ + \sum_{i=1}^{I} \sum_{p=1}^{P} \sum_{t=1}^{T} M \cdot (AGA_{i,t} + ADP_{i,p,t}) \]

\[ - \sum_{i=1}^{I} \sum_{p=1}^{P} \sum_{t=1}^{T} TS_{i,p,t} \cdot QSP_{i,p,t} \]

\[ \text{CONSTRAINTS LIMITING RAW MILK SUPPLY AT FARM LEVEL} \]

(2) Grade A milk supply balance \( \forall i \in I, \forall t \in T \)

\[ \sum_{j=1}^{J} (XGA_{i,j,c,t} + XSP_{i,j,p \in J,c,t}) + SGA_{i,c+1,t} \leq QGA_{i,c,t} + AGA_{i,t} \quad c \in \{ \text{Class I} \} \]

\[ \sum_{j=1}^{J} XGA_{i,j,c,t} + SGA_{i,c+1,t} \leq SGA_{i,c,t} \quad c \in \{ \text{Class II, III} \} \]
\[ \sum_{j=1}^{J} XGA_{i,j,c,t} \leq SGA_{i,c,t} \quad c \in \{ \text{Class IV} \} \]

(3) Grade B milk supply balance \( \forall i \in I, \forall t \in T \)
\[ \sum_{j=1}^{J} XGB_{i,j,c^M_{t+1}} + SGB_{i,c^M_{t+1},t} \leq SNB_{i,c,t} + QGB_{i,c,t} \quad c^M \in \{ \text{Class III} \} \]
\[ \sum_{j=1}^{J} XGB_{i,j,c^M_{t+1}} \leq SGB_{i,c^M_{t+1},t} \quad c^M \in \{ \text{Class IV} \} \]

(4) Unregulated milk supply balance \( \forall i \in I, \forall t \in T \)
\[ XUF_{i,i,t} + SNB_{i,c,t} \leq QGU_{i,c,t} \]
\[ XUF_{i,i,t} \leq \alpha \times QGU_{i,c,t} \]

\section*{Constraints Balancing Raw Milk at a Processing Plant}

(5) Classified milk supply balance \( \forall i \in I, \forall t \in T \)
\[ QRP_{i,c,t} \leq \sum_{j=1}^{J} (XGA_{j,i,c,t} + XSP_{j,i,c,t}) + XUF_{i,i,t} \quad c \in \{ \text{Class I} \} \]
\[ QRP_{i,c,t} \leq \sum_{j=1}^{J} XGA_{j,i,c,t} \quad c \in \{ \text{Class II} \} \]
\[ QRP_{i,c^M_{t+1}} \leq \sum_{j=1}^{J} (XGA_{j,i,c^M_{t+1}} + XGB_{j,i,c^M_{t+1}}) \quad c^M \in C^M \]

(6) Classified milk demand balance \( \forall i \in I, \forall t \in T \)
\[ \sum_{l=1}^{L} \sum_{r=1}^{R} QRI_{l,c,t} \times QDP_{i,l,r,t} \leq QRP_{i,c,t} \quad c \in \{ \text{Class I, III, IV} \} \]
\[ \sum_{l=1}^{L} \sum_{r=1}^{R} QRI_{l,c,t} \times QDP_{i,l,r,t} + \sum_{m=1}^{M} QRM_{i,c,m,t} \leq QRP_{i,c,t} \quad c \in \{ \text{Class II} \} \]

(7) Supplying milk balance \( \forall i \in I, \forall t \in T, c \in \{ \text{Class I} \} \)
\[ \sum_{j=1}^{J} XSP_{i,p,j,c,t} \leq \sum_{j=1}^{J} XSP_{j,i,c,t} \]

(8) Maximum capacity constraints \( \forall i \in I, \forall t \in T, \forall c \in C \)
\[ QRP_{i,c,t} \leq CP_{i,c} \]
 Constraints Balancing Products at a Processing Plant

(9) Intermediate product demand balance \( \forall i \in I, \forall t \in T, \forall c \in C \)
\[
\sum_{i=1}^{L} \sum_{r=1}^{R} QPI_{i,p,r} \cdot QDP_{i,l,r,t} + \sum_{m=1}^{M} QPM_{i,p,m,t} \leq \sum_{j=1}^{J} XPI_{j,i,p,t}
\]

(10) Volume balance at blending problem \( \forall i \in I, \forall t \in T, \forall m \in M \)
\[
QMI_{i,m,t} \leq QRM_{i,c=2,m,t} + \sum_{p=1}^{P} QPM_{i,p,m,t}
\]

(11) Mixed product demand balance \( \forall i \in I, \forall t \in T, \forall m \in M \)
\[
\sum_{i=1}^{L} \sum_{r=1}^{R} QMI_{i,m,l,r} \cdot QDP_{i,l,r,t} \leq QMI_{i,m,t}
\]

(12) Component balance at blending problem \( \forall i \in I, \forall t \in T, \forall m \in M, \forall c \in CH \)
\[
\overline{RCH}_{c} \cdot QRM_{i,c=2,m,t} + \sum_{p=1}^{P} \overline{PCH}_{p,c} \cdot QPM_{i,p,m,t} = \overline{MCH}_{m,c} \cdot QMI_{i,m,t}
\]

(13) Maximum dry whey contents on blending problem \( \forall i \in I, \forall t \in T, \forall m \in M \)
\[
\sum_{p'=1}^{P} QPM_{i,p',p,m,t} \leq \overline{MAW}_{m} \cdot QMI_{i,m,t}
\]

(14) Product supply balance \( \forall i \in I, \forall t \in T, \forall p \in P \)
\[
XFP_{i,p,t} + \sum_{j=1}^{J} (XPI_{j,p,t} + XPD_{j-p,t}) + \sum_{j=1}^{J} XSA_{i,j,p'} \leq \sum_{i=1}^{L} \sum_{r=1}^{R} QPO_{p,l,r} \cdot QDP_{i,l,r,t}
\]

Constraints Related to Stock Levels

(15) \( \forall i^S \in I^S, \forall t \in T, \forall p^S \in P^S \)
\[
\overline{QSP}_{i^S,p^S,t} + \sum_{j=1}^{J} \overline{XSR}_{i,j,p^S,t} = \overline{QBS}_{i^S,p^S,t} + \sum_{j=1}^{J} \overline{XSA}_{i,j,p^S,t} + \overline{QSP}_{i^S,p^S,t-1}
\]

\[
\overline{MIS}_{i^S,p^S,t} \leq \overline{QSP}_{i^S,p^S,t}
\]

Constraints Related to Final Product Demand

(16) \( \forall i \in I, \forall t \in T, \forall p \in P \)
\[
\sum_{j=1}^{J} XPD_{j,i,p,t} + \sum_{j=1}^{J} XSR_{j,i,p,t} \leq \overline{QPD}_{i,p,t} + ADP_{i,p,t}
\]
REAL-WORLD CONSTRAINTS

(17) Class I milk shipped through supply plants \( \forall a^P \in A^P, \forall t \in T \)

\[
\sum_{j=1}^{J} \sum_{i \in (I \cap A^T)} \bar{XSP}_{j,i,c=1,t} = \sum_{j=1}^{J} \sum_{i \in (I \cap A^T)} \bar{PS}_{a,c} XGA_{j,i,c=1,t}
\]

(18) Minimum capacity use by Marketing Order area \( \forall a^F \in A^F, \forall t \in T, \forall c \in C \)

\[
\sum_{i \in (I \cap A^T)} \bar{CP}_{i,c} \times \frac{MCU_{a,c}}{a^F} \leq \sum_{i \in (I \cap A^T)} QRP_{i,c,t}
\]

(19) Minimum restriction on Class I and Class II supply \( \forall i \in I \cap F, \forall t \in T \)

\[\text{load} \leq \sum_{j=1}^{J} \sum_{c=1}^{2} XGA_{i,j,c,t}\]